ADVANCES IN MATERIALS SCIENCE, Vol. 22, No. 4 (74), December 2022

DOI: 10.2478/adms-2022-0022

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TIME-PERIODIC THERMAL BOUNDARY EFFECTS ON POROUS MEDIA SATURATED WITH NANOFLUIDS: CGLE MODEL FOR OSCILLATORY MODE

ABSTRACT

The stability of nonlinear nanofluid convection is examined using the complex matrix differential operator theory. With the help of finite amplitude analysis, nonlinear convection in a porous medium is investigated that has been saturated with nanofluid and subjected to thermal modulation. The complex Ginzburg-Landau equation (CGLE) is used to determine the finite amplitude convection in order to evaluate heat and mass transfer. The small amplitude of convection is considered to determine heat and mass transfer through the porous medium. Thermal modulation of the system is predicted to change sinusoidally over time, as shown at the boundary. Three distinct modulations IPM, OPM, and LBMOhave been investigated and found that OPM and LBMO cases are used to regulate heat and mass transfer. Further, it is found that modulation frequency (ω_f varying from 2 to 70) reduces heat and mass transfer while modulation amplitude ($\delta_1$varying from 0.1 to 0.5) enhances both.

Keywords:*Nanofluid convection; transport analysi; CGLE; numerical performance; thermal modulation; nonlinear theory*

INTRODUCTION

The study of fluid mechanics is essential for both domestic and industrial demands. Nanofluids in particular are becoming more important because of their useful applications in engineering and thermal science research. It is clear that nanofluids efficiently transfer heat and mass tranasfer as compared to base fluids. A huge amount of fluid or material properties are changing in time and space as a result of their abnormal behavior. The unpredictable nature of these fluid fluxes must be considered. The convective instability of nanofluids has been the subject of various investigations in the literature. The fundamental studies of nanofluids are reported by Choi [1], Masuda et al. [2], Chen [3], Eastman et al. [4], Das et al. [5] Buongiorno and Hu [6]. Oyelakin et al. [7] is investigated the flow of a 3 dim nanofluid across a stretching sheet using Buongiorno model. They discovered that the rate of heat transfer in Brownian motion is extremely low. The above studies reported on heat mass transfer in nanofluid natural convection.

However, some metals have a comparatively high thermal conductivity compared to ordinary fluids. The basic concept behind these nanofluids was to suspend them have them behave like a fluid, and give them metal like thermal conductivity. By adding the nano particles to regular fluids the thermal conductivity may be enhanced by 15–40%.In order to make nanofluids, nanoparticles of metal, metallic oxides, or nonmetallic oxides are suspended in common base fluids including water, ethylene glycol, and engine oils. The majority of these heat-exchanging situations are found in modern engineering and research, including biomechanics, geophysical problems, spinning machinery like nuclear reactors, food, and chemical processing, and the petroleum industry. The variety of nanofluid applications is due to their enhancing nature of heat and mass transfer with mixed low concentration nano-sized particles. Nanofluids control the transport processes that can be used in drug delivery systems.

Eastman et al. [8] investigated the effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. It is found that a strong effect of particle shape on thermal conductivity, but no effect of either particle size or particle thermal conductivity. Rea et al. [9] reported studies of temperature-dependent thermophysical properties of nanofluids, including pressure loss and heat transfer coefficient. In the context of Brownian diffusion and nanofluid thermophoresis, Buongiorno [10] observed a convective transport phenomenon of nanofluids. Tzou [11, 12] found that turbulence flow increases the energy bearing capacity of nanofluids, which could lead higher heat transfer than the increase of the thermal conductivity alone. When the Darcy model is used for a nonlinear stability study, Nield et al. [13] investigated the impacts of viscous dissipation and pressure work resulting in a reduced heat transfer when the Darcy R_0 number is substituted by the superadiabatic Darcy R_0. The defect of LTN on the Onset of convection in a porous medium was reported by Kuznetsov and Nield [14]. Also applying the Brinkman model, the study of thermal instability in a porous nanofluid layer was investigated by Kuznetsov et al. [15].

In a rotating porous medium, Bhadauria et al. [16] investigated the nonlinear analysis of nanofluid convection. They have solved nonlinear system of coupled equation numerically and evaluated haet mass transfer results. It is found that rotational effects are control transfer phenomenon. Agarwal et al. [17] examined the rotational and anisotropy effects of porous media on nanofluid convection as part of a series of studies on this topic. It is found that anisotropy and rotation has dual nature of stability analysis. Also she reported different boundary conditions more realistic approach on nanoconvection [18]. Rana et al. [19] investigated multi nanoconvection in porous media with rotation. While calculating critical R0 and Nu discussed stability onset convection and transfer analysis. Agarwal et al. [20] studied LTN on binary nano-convection with (Al_2O_3) porous saturated layer. Most of the works reported on nanoconvection in fluid or porous layer are considered without modulation. Umavathi et al. [21] investigated the chemical reaction effect on nanofluid convection in a porous layer with the help of stability analysis. It was found that the onset convection is delayed with a growth of the viscosity ratio. The effect of g-jitter on double-diffusive oscillatory convection in a viscoelastic fluid layer was reported by Kiran [22]. The complex Ginzburg-Landau equation was applied to measure the heat and mass transfer in terms of finite complex amplitude. The effect of temperature dependant viscosity on weak nonlinear Rayleigh-Benard convection under rotation speed modulation was reported by Manjula and Kiran [23].

In this paragraph other nonlinear models is discussed to know the importance of nonlinear problems in real life. Ibrahim et al. [24] uses the Kellar box approach to discuss the effects of viscous dissipation, melting over a stretching sheet, a chemical reaction on williamson and maxwell nanofluids. Ajibade et al. [25] used the DTM approach to study suction and injection on a vertical channel. They discovered that skin friction, velocity, and suction both reduce the transfer phenemonon. L$\ddot{u}$ et al. [26] investigated the appropriate control strategies of COVID-19 to minimize the control cost and ensure the normal operation of society under the premise of containing the epidemic. Yin et al. [27] presented some suggestions to prevent and control of COVID-19. The newly proposed ECIMM algorithm can be used to deal with the doubly interval-censored data model appearing in various fields. Yin et al. [28] investigated different types of N soliton solutions to the (3+1) dimensional nonlinear evolution equation for the shallow-water waves using Hirota bilinear method. To describe certain wave process in oceanography, acoustics or hydrodynamics Zhao et al. [29] investigated the extended coupled (2+1) dimensional Burgers system with the Riccati projective equation method. Liu et al. [30] found the numerical and rogue wave solutions for coupled nonlinear Schr$\ddot{o}$dinger model to explore the influence of external potential on option price.

Venezian [31] and Gresho et al. [32] were the first who started works on thermal and gravity modulation for linear models. Approaching the perturbation analysis they have derived shift in the critical R_0 as a function of wavenumber. Their analysis reported that while suitably adjusting Ω-frequency and δ-amplitude of modulation one can control stability analysis. The impact of temperature and gravitation modulation on the initiation of thermal instability in a couple-stress layer were both studied by Malashetty et al. [33]. Shu et al. [34] and Rogers et al. [35] studied the complex ordered patterns of convection and experimental measurements of convection with numerical simulations under gravity modulation. The effect of quasiperiodic g-jitter on the stability was examined by Boulal et al. [36]. Umavathi [37] used a linear stability analysis to investigate the impact of heat modulation on the onset porous nano convection. It has been shown that time-periodic variations in the wall temperatures can be used to regulate stability analysis.

The Darcy model was utilized by Bhadauria et al. [38] to investigate nonlinear thermal convection in a porous medium saturated with nanofluid. The above problem was further developed by Bhadauria et al. [39, 40] for the internal heating effect of gravity-driven nanofluid convection. The effect of g-jitter on nonlinear thermal convection in a porous medium saturated with viscoelastic nanofluid was discussed by Kiran [41]. Kiran et al. [42, 43] revealed the results of the interactions between internal heating and centrifugal force on nano-convection under thermal modulation. Kiran et al. [44] studied porous nanoconvection using a regulated heat source and sink analysis. It is found that internal heat modulation is more effective than other modulations. The studies of Bhadauria et al. [45, 46] and Kiran et al. [47] are a collection of works that uses the Fourier series expansion of physical variables to explain Newtonian and non-Newtonian fluid convection under modulation. The investigations from the above literature showed that modulation can be applied in various ways to influence both instability and transport phenomena.

The complex Ginzburg-Landau model for nano-convection with modulation has not yet been reported in any publications. As a result, we discuss the results of our research using the complex Ginzburg-Landau equation for nanoconvection. The goal of the current work is to investigate the effect of plate modulation on nanofluid convection. The perturbation approach is used to derive finite amplitude analytically and numerically. This article examines the consequences of change in three different temperature profiles: IPM, OPM, and LBM. The comparison of three different modulations have been reported.

GOVERNING EQUATIONS

A porous layer saturated with a nanofluid is considered between two horizontal walls lying at $z = 0$ and $z = d$ respectively. The layer is fully impermeable and thermally conductive. It is infinitely extended in x and y directions and lying lower plate at origin. The temperature of lower plate and upper plate are given by T_h and T_c.

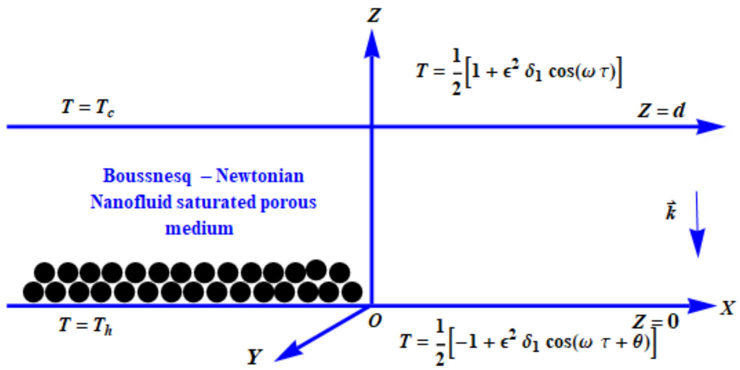

Fig.1. Physical configuration of the problem

It is observed that $T_h > T_c$ which indicates lower plate is hotter than upper wall. The illustration of the issue is shown in Figure 1. According to Buongiorno [10], Kuznetsov and Nield [14], Kiran and Manjula [44] the governing equations are taken and employed Oberbeck-Boussinesq prediction:

$$\nabla \cdot v_D = 0, \tag{1}$$

$$\frac{\rho_f}{\delta}\frac{\partial v_D}{\partial \tau} + \nabla p = -\frac{\mu}{K} v_D + [\phi \rho_p + \{\rho(1 - \beta(T - T_c))\}(1 - \phi)]\vec{g} \tag{2}$$

$$\gamma \frac{\partial T}{\partial \tau} + \frac{(\rho c)_f}{(\rho c)_m} v_D \cdot \nabla T = \frac{1}{(\rho c)_m} k_m \nabla^2 T + \frac{\delta (\rho c)_p}{(\rho c)_m} D_B \nabla \phi \cdot \nabla T, \tag{3}$$

$$\frac{\partial \phi}{\partial \tau} + \frac{1}{\delta} v_D \cdot \nabla \phi = D_B \nabla^2 \phi, \tag{4}$$

(5)

here v_D is the Darcy velocity, $\nu = \mu/\rho_f$ Kinematic viscosity and $\gamma = \frac{(\rho c)_m}{(\rho c)_f}$ heat capacity ratio. Assuming that the temperature and the volumetric fraction of the nanoparticles are constant at the stress-free boundaries then the boundary conditions on T and ϕ as follows:

$$(v, T, \phi) = (0, T_h, \phi_0) \text{at } z = 0, \tag{6}$$

$$(v, T, \phi) = (0, T_c, \phi_1) \text{at } z = d, \tag{7}$$

The following externally applied thermal boundary conditions are taken in this paper (Venezian [14], Bhadauria and Kiran [45, 46])

$$(T_{z=0}, T_{z=d}) = (\tfrac{1}{2}[1 + \epsilon^2 \delta_1 \cos(\omega_f t)], \tfrac{1}{2}[-1 + \epsilon^2 \delta_1 \cos(\omega_f t + \theta)]) \tag{8}$$

where δ_1 is the amplitude of modulation, Ω is the amplitude frequency of modulation and θ is the phase difference. The following non-dimensional variables are taken:

$$(X^*) = X/d, \tau^* = \tau k_T/\gamma d^2, (v_d^*) = v_d d/k_T, p^* = pK/\mu k_T, \phi^* = \frac{\phi-\phi_0}{\phi_1-\phi_0} \text{ and } T^* = \frac{T-T_c}{T_h-T_c}, \text{ where } k_T = \frac{k_m}{(\rho c)_f}, \gamma = \frac{(\rho c_p)_m}{(\rho c_p)_f}.$$

After using the above the governing equations are (after dropping the asterisk for simplicity):

$$\nabla \cdot v = 0 \tag{9}$$

$$\frac{1}{Va}\frac{\partial v}{\partial \tau} = -\nabla p - v - \mathrm{g}_m(Rm - R_T\ T + Rn\phi)\hat{e}_z \tag{10}$$

$$v \cdot \nabla T = -\gamma\frac{\partial T}{\partial \tau} + \nabla^2 T + \frac{N_B}{Le}\nabla\phi \cdot \nabla T, \tag{11}$$

$$v \cdot \nabla\phi = -\frac{\partial \phi}{\partial \tau} + \frac{1}{Le}\nabla^2\phi, \tag{12}$$

$$(v, T, \phi) = (0,1,0), \text{at } z = 0, \text{and} (v, T, \phi) = (0,0,1) \text{at } z = 1 \tag{13}$$

The nanofluid is considered to be at rest in its basic state therefore, its quantities will only vary in the z-direction, as shown by the following equations:

$$v = 0, p = p_b(z), \quad \phi = \phi_b(z). \quad T = T_b(z). \tag{14}$$

The following equation (8) is obtained while using the Eq.(14) in Eqs.(9-11):

$$d^2T_b/dz^2 + N_B Le^{-1}\frac{d\phi_b}{dz}\frac{dT_b}{dz} = 0 \tag{15}$$

According to the studies of Tzou [11, 12] and Kuznetsov & Nield [16], the basic state solutions (using the Eq.(8, 15)) are given by [31, 34-37]:

$$\frac{\partial T_b}{\partial z} = -1 + \epsilon^2\delta_1[f_2(z,t)] \tag{16}$$

$$\phi_b = z. \tag{17}$$

where

$$f_2 = \mathrm{Re}[f_1(z)Exp[-i\omega_f t], \tag{18}$$

$$f_1(z) = [F(\lambda)e^{\lambda z} + F(-\lambda)e^{-\lambda z}], F(\lambda) = \frac{\lambda}{2}\frac{(e^{-i\theta} - e^{-\lambda})}{(e^{\lambda} - e^{-\lambda})} \text{and } \lambda = (1-i)\sqrt{\frac{\omega_f}{2}}.$$

NONLINEAR THERMAL STABILITY

The perturbation technique is used to find the approximate solutions to nonlinear system of equations. Let us superimpose the following perturbations onto the basic state:

$$X = X_b + X' \tag{19}$$

where X stands for physical variable and it takes X=(v, p, T, ϕ). The following system of coupled equations are obtained by replacing the above expression Eq.(19) in Eqs. (9-12):

$$\left(Va^{-1}\frac{\partial}{\partial s}+1\right)\nabla^2\psi = Rn\frac{\partial \phi}{\partial x}g_m\vec{k} - R_T\frac{\partial T}{\partial x}g_m\vec{k} \tag{20}$$

$$-\frac{\partial T_b}{\partial z}\frac{\partial \psi}{\partial x} + (\gamma\partial/\partial s - \nabla^2)T = \frac{\partial(\psi,T)}{\partial(x,z)} \tag{21}$$

$$-\delta^{-1}\frac{\partial \psi}{\partial x} + \left(\frac{\partial}{\partial s} - \frac{1}{Le}\nabla^2\right)\phi = \frac{\partial(\psi,\phi)}{\partial(x,z)} \tag{22}$$

where $Va = \frac{\delta Pr}{Da}$ Prantdl-Darcy number, $Le = \frac{k_T}{D_B}$ Lewis number, $N_B = \delta(\rho c)_p {(\rho c)_f}^{-1}(\phi_1 - \phi_0)$ increment of modified density, $Ra = \frac{\rho g_0 \beta K d(T_h - T_c)}{\mu k_T}$ thermal Rayleigh-Darcy number, $Rm = \frac{[\rho_p\phi_0 + \rho(1-\phi_0)]g_0 K d}{\mu k_T}$, basic nano-density Rayleigh number, $Rn = \frac{(\rho_p - \rho)(\phi_1 - \phi_0)g_0 K d}{\mu k_T}$, concentration Rayleigh number. To analyze oscillatory convection of the system slow variation of time is taken according to Bhadauria et al. [45, 46] and [48-51], Rajib et al. [54]. The above system of Eqs.(20-22) are solved using stress-free, isothermal and iso-nano-concentration boundary conditions:

$$(\psi = 0, T = 0, \phi = 0)\text{at} \quad z = 0,1 \tag{23}$$

HEAT TRANSFER AND THE FINITE AMPLITUDE EQUATION FOR OSCILLATORY INSTABILITY

The following asymptotic expansions are now introduced in Eq. (20–22) to simplify nonlinearity and evaluate solutions:

$$R_T = R_{0c} + \epsilon^2 R_2 + \epsilon^4 R_4 + \ldots,$$
$$X = \epsilon X_1 + \epsilon^2 X_2 + \epsilon^3 X_3 + \ldots,, \tag{24}$$

where, $X = (\psi, T, \phi)$ and $R_{0c}$is the critical onset Rayleigh number. The system is now solved for various orders of ϵ. The critical R_{0c} marks the onset convection in the absence of modulation. Here the system is solved for various orders of ϵ. The lowest system takes the form:

$$M\begin{bmatrix}\psi_1\\ T_1\\ \phi_1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \tag{25}$$

where:
$$M = \begin{bmatrix}\left(Va^{-1}\frac{\partial}{\partial \tau}+1\right)\nabla^2 & R_{0c}\frac{\partial}{\partial x} & -Rn\frac{\partial}{\partial x}\\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & (\gamma\partial/\partial\tau - \nabla^2) & 0\\ -\delta^{-1}\frac{\partial}{\partial x} & 0 & \left(\frac{\partial}{\partial \tau} - \frac{1}{Le}\nabla^2\right)\end{bmatrix}$$

The solutions of the lowest order system are given by:

$$(\psi_1, T_1, \phi_1) = [(\mathbb{M}(s)e^{i\omega\tau} + \overline{\mathbb{M}}(s)e^{-i\omega\tau})\sin ax, (\mathbb{N}(s)e^{i\omega\tau} + \overline{\mathbb{N}}(s)e^{-i\omega\tau})\cos ax$$
$$(\mathbb{O}(s)e^{i\omega\tau} + \overline{\mathbb{O}}(s)e^{-i\omega\tau})\cos ax]\sin \pi z (26)$$

The above are considered according to the Eq.(28). The following relation is derived for undetermined amplitudes:

$$(\mathbb{N}(s),\mathbb{O}(s))=(-\frac{a\mathbb{M}(s)}{c+i\omega\gamma},-\frac{a\mathbb{M}(s)}{\delta(\frac{c}{Le}+i\omega)}) \quad (27)$$

where $c=a^2+\pi^2$. The critical Rayleigh number and the related wavenumber are given by:

$$R_0=\frac{RnLe(\omega^2\gamma Le+c^2)}{\delta(\omega^2Le^2+c^2)}+\frac{c}{a^2}(c-\omega^2\gamma Va) \quad (28)$$

The oscillatory frequency of the disturbances is given by:

$$\omega^2=\frac{Rna^2(Le-\gamma)}{\delta(\gamma+cVa)}-\frac{c}{Le} \quad (29)$$

It is observed that the overstability for a specific wavenumber a can only exist if the following inequality is true ($Le>\gamma$). The system nonlinearity is revealed itself in the second order. The terms of this case are given by:

$$M\begin{bmatrix}\psi_2\\T_2\\\phi_2\end{bmatrix}=\begin{bmatrix}Rh_{21}\\Rh_{22}\\Rh_{23}\end{bmatrix} \quad (30)$$

$(Rh_{21},Rh_{22},Rh_{23})=(0,J(\psi_1,T_1),J(\psi_1,\phi_1))$(31)

where J stands for Jacobian. According to the analysis of Bhadauria et al. [49-51] the terms of T_2,ϕ_2 takes the following form:

$$(T_2,\phi_2)=\{(T,\phi)_{20}+(T,\phi)_{22}e^{2i\omega\tau}+\overline{(T,\phi)}_{22}e^{-2i\omega\tau}\}\sin 2\pi z \quad (32)$$

where $(T,\phi)_{22}$ and $(T,\phi)_{20}$ are the temperature fields having the terms of frequency 2ω and independent of fast time scale. The solutions are given by (based on first order solutions)

$$(T_{20},\phi_{20})=(\frac{a}{8\pi}\{\mathbb{M}(s)\overline{\mathbb{N}}(s)+\overline{\mathbb{M}}(s)\mathbb{N}(s)\},\frac{aLe}{8\pi}\{\mathbb{M}(s)\overline{\mathbb{O}}(s)+\overline{\mathbb{M}}(s)\mathbb{O}(s)\}) \quad (33)$$

and

$$(T_{22},\phi_{22})=(\frac{\pi a}{8\pi^2+4i\omega}\mathbb{M}(s)\mathbb{N}(s),\frac{\pi aLe}{8\pi^2+4i\omega Le}\mathbb{M}(s)\mathbb{O}(s)) \quad (34)$$

Heat transfer coefficient, the Nusselt number $Nu(s)$ is given by:

$$Nu=1+\left(\frac{a^2c}{2(c^2+\omega^2\gamma^2)}+\frac{2\pi^2a^2}{4\sqrt{(c^2+\omega^2\gamma^2)}\sqrt{(4\pi^2+\omega^2\gamma^2)}}\right)|\mathbb{M}(s)|^2 \quad (35)$$

Mass transfer coefficient, the concentration Nusselt number $Nu_c(s)$ is given by:

$$Nu_c(s)=1+\left(\frac{a^2Le\Delta_1}{2\delta}+\frac{a^2Le\pi^2}{2\delta}\frac{\sqrt{(Le^2c^2+\omega^2\gamma^2Le^2)}}{\sqrt{4\pi^2+\omega^2Le^2}\sqrt{(c^2+\omega^2\gamma^2)}\sqrt{c^2+\omega^2\gamma^2}}\right)|\mathbb{M}(s)|^2 \quad (36)$$

where $\Delta_1=\frac{Lec(c^2-\omega^2Le\gamma)+\omega^2c(Le+\gamma)Le\gamma}{(c^2-\omega^2Le\gamma)^2+\omega^2c^2(Le+\gamma)^2}$.

The third order system is in the following form:

$$M\begin{bmatrix}\psi_3\\ T_3\\ \phi_3\end{bmatrix}=\begin{bmatrix}Rh_{31}\\ Rh_{32}\\ Rh_{33}\end{bmatrix} \tag{37}$$

where

$$Rh_{31}=-\frac{1}{Va}\frac{\partial}{\partial s}(\nabla^2\psi_1)-R_2\frac{\partial T_1}{\partial x}-R_{0c}\frac{\partial T_2}{\partial x}+Rn\frac{\partial \phi_2}{\partial x}, \tag{38}$$

$$(Rh_{32},Rh_{33})=(-\gamma\frac{\partial T_1}{\partial s}+\frac{\partial \psi_1}{\partial x}\frac{\partial T_2}{\partial z}++\delta_1 f_2\frac{\partial \psi_1}{\partial x},-\frac{\partial \phi_1}{\partial s}+\frac{\partial \psi_1}{\partial x}\frac{\partial \phi_2}{\partial z}) \tag{39}$$

The terms of Rh_{31}, Rh_{32} and Rh_{33} are simplified with the help of first and second order solutions. The Ginzburg-Landau equation for oscillatory convection with time-periodic coefficients is obtained under the solvability condition (Manjula and Kiran [23], Bhadauria et al. [45, 46] and [49-51]):

$$\frac{\partial \mathbb{M}(s)}{\partial s}-Q_1\mathbb{M}(s)+Q_2|\mathbb{M}(s)|^2\mathbb{M}(s)=0 \tag{40}$$

where the coefficients $Q_1=F_1F_2,\ Q_2=F_1F_3$, and the factors F_1,F_2,F_3 are given by:

$$F_1=\frac{c}{Va}+\frac{a^2R_0}{(c+i\omega\gamma)^2}-\frac{a^2RnLe(cLe+i\omega Le\gamma)}{\delta(c+i\omega\gamma)(c+i\omega Le)^2}$$

$$F_2=\frac{a^2R_0}{(c+i\omega\gamma)}-\frac{2a^2R_0}{(c+i\omega\gamma)}\delta_1 I_1,$$

$$F_3=\frac{R_0}{(c+i\omega\gamma)}\left(\frac{a^4c}{4(c^2+\omega^2\gamma^2)}+\frac{a^4\pi^2}{2(2\pi^2+i\omega\gamma)(c+i\omega\gamma)}\right)-\frac{a^4LeRn\Delta_1}{4\delta}+\frac{a^4\pi^2Le^2Rn}{\delta(c+i\omega Le)(8\pi^2+4i\omega Le)(c^2-\omega^2Le\gamma+i\omega c(Le+\gamma))}$$

Writing $\mathbb{M}(s)$ in the phase-amplitude form:

$$\mathbb{M}(s)=|\mathbb{M}(s)|e^{i\phi} \tag{41}$$

The following equation is obtained while substituting the Eq.(41) in Eq.(40) for the amplitude $|\mathbb{A}(s)|$:

$$\frac{\partial|\mathbb{M}(s)|^2}{\partial s}-2p_r|\mathbb{M}(s)|^2+2l_r|\mathbb{M}(s)|^4=0 \tag{42}$$

$$\frac{\partial(ph(\mathbb{M}(s)))}{\partial s}=p_i-l_i|\mathbb{M}(s)|^2 \tag{43}$$

where $Q_1=p_r+ip_i,\ Q_2=l_r+il_i$ and $ph(.)$ represents the phase shift.

RESULTS

The effect of thermal modulation on nano-porous convection has been investigated for oscillatory mode. To analyze the heat/mass transport, a weak nonlinear stability analysis is used for the Darcy model while taking the top-heavy porous layer into account. It is fact that linear theory reports stability analysis while nonlinear theory reports transport phenomena (Umavathi [20]). In this paper thermal modulation is considered for either strengthening or diminishing the convective transport phenomena.

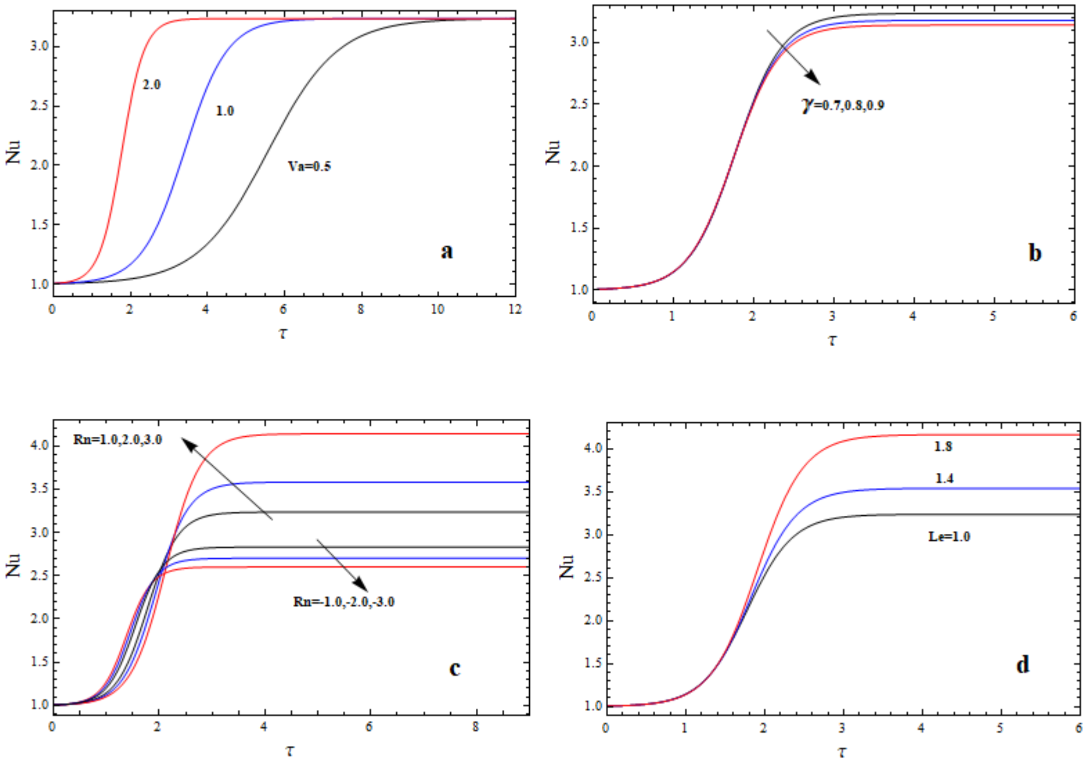

Fig. 2. Heat transfer results for different values of system parameters (IPM case)

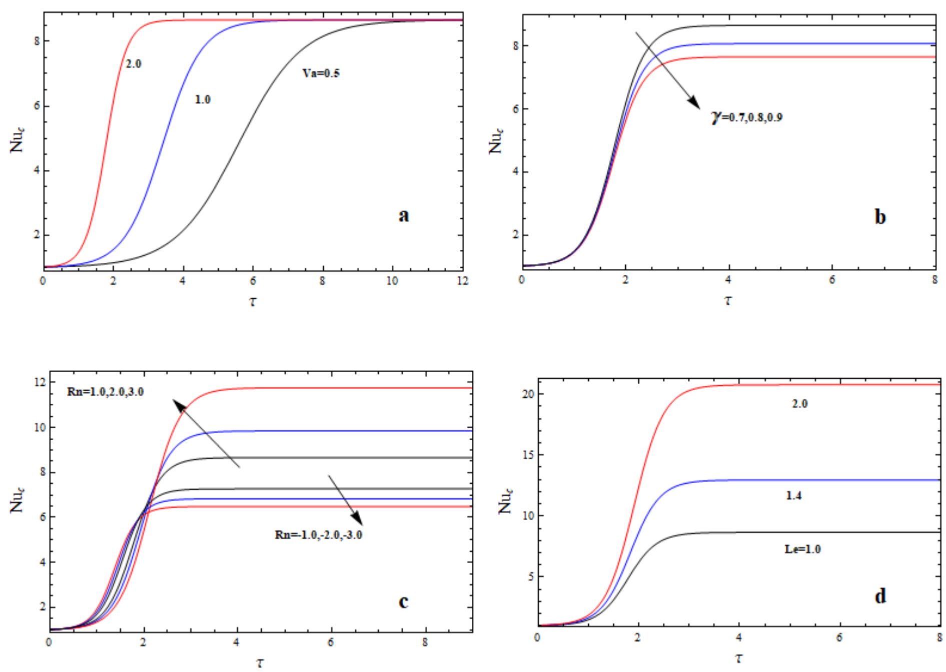

Fig. 3. Nano concentration transfer results for different values of system parameters (IPM case)

Thermal modulation is considered since the external restrictions of convection are significant in the study of thermal instability. Buongiorno [6] claims that Le is significant for the majority of the nanofluids that has been studied so far. In our calculations R_{0c} is calculated taking the moderate Le values (Bhaduria et al. [31]) for existing oscillatory convection. Three different thermal boundary conditions are taken to see the modulation effect on Nu and Nu_c. The following different modulation conditions are considered:

1. IPM -both plates are modulated in the same manner $\theta = -I\infty$
2. OPM -both plates are modulated in opposite manner $\theta = \pi$
3. LBM -only lower plate modulated $\theta = 0$.

Weak nonlinear stability analyses is used to examine the impact of TM (temperature modulation) on the Darcy convection. It has been proposed that the modulation is of order $O(\epsilon^2)$ which exhibits modulation with a low amplitude. Our presumption makes it considerably simpler than the Lorenz model to arrive amplitude equation. It is widely known that the construction of a nonlinear theory used to assess heat and mass transfer and it is not permitted by the linear theory. In this study the TM effect has been taken into account to either enhance or diminish heat mass transport. Only linear models with finite terms of the Fourier series were established in the previous decade. For nonlinear situations of nanofluid convection no CGLE model has yet been implemented. As a result our paper implemented CGLE model to describe the modulated nano-convection.

The results of our study are presented in Figures 2 to 7 and discussed the heat and mass transfer results. Figure 8 shows stability analysis curves. In our investigation the following parameters are affect the convective problem: $Va, Rn, Le, \gamma, \delta, \delta_1$. The first 4 parameters are related to the fluid layer and the next 2 parameters concern convection control. Since the porous layer isn't thought to be particularly viscous, only modest values of Va are used in our calculations. The parameter δ_1 and ω_f are the external mechanism for regulating convection. Since the fluid layer isn't considered to be particularly viscous, only modest quantities of Va are used in our calculations. The value of δ_1 is thought to be minimal due to the small amplitude modulation.

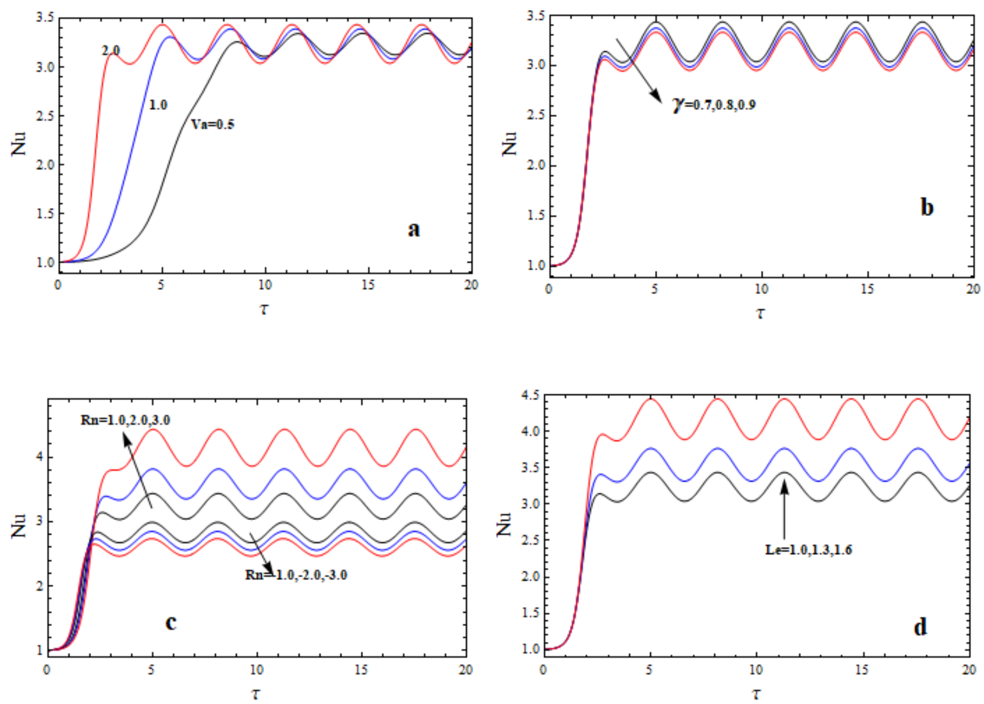

Fig. 4. Heat transfer results for different system parameters (OPM case)

In our investigation TM is assumed to be of low-frequency order, since heat mass transfer is most efficient at low-frequency ranges. Based on the CGLE a weak nonlinear stability was carried out. The thermal Nusselt number and the concentration Nusselt number are determined as a functions of time and other system variables. Through a finite amplitude $\mathbb{M}$ the transport phenomena heat and mass transfer in terms of Nu and Nu_c are numerically evaluated. The reader may observe that the evaluation of $\mathbb{M}$ is difficult because the modulation varies over a slow time. Additionally it is noted that the CGLE may be solved analytically and transforms into a Bernoulli DE without modulation.

Table 1. *Readings of R0c, kc for Le, Rn, Va, δ and γ*

Variable	R0c (st)	a_c (st)	R0c (osc)	a_c(osc)
Le	-	-	.	
2.0	55	3.14	75	2.52
3.0	63	3.14	60	2.59
4.0	71	3.14	52	2.64
Rn	-	-	.	
2.4	55	3.14	75	2.52
3.4	62	3.14	67	2.516
4.4	69	3.14	59	2.510
Va	-	-	.	
1.5	55	3.14	75	2.52
2.0	55.4	3.14	91	2.47
3.0	55.4	3.14	109	2.43

δ	-	-	.	
0.3	55.4	3.14	75	2.52
0.4	51.47	3.14	113	2.43
0.5	49.07	3.14	115	2.435

γ	-	-	.	
0.1	55	3.14	75	2.52
0.2	51.47	3.14	192	2.34
0.3	49.47	3.14	274	2.30

In Figures 2 to 7 the impact of TM on heat/mass transport are shown. The purpose of Va is to promote heat and concentration transport for small values of time, and comparable effects may be seen over time (given in Figures 2a and 3a for IPM case). Effect of Va is of quite natural to enhancement of transport phenomenon. For nono fluid cases one may observe the related studies of Va effects by Umavathi [37], Bhadauria and Agarwal [16, 53] and Agarwal et al. [17, 18, 20]. It is observed that lower values of Va show variations in heat/concentration transport.

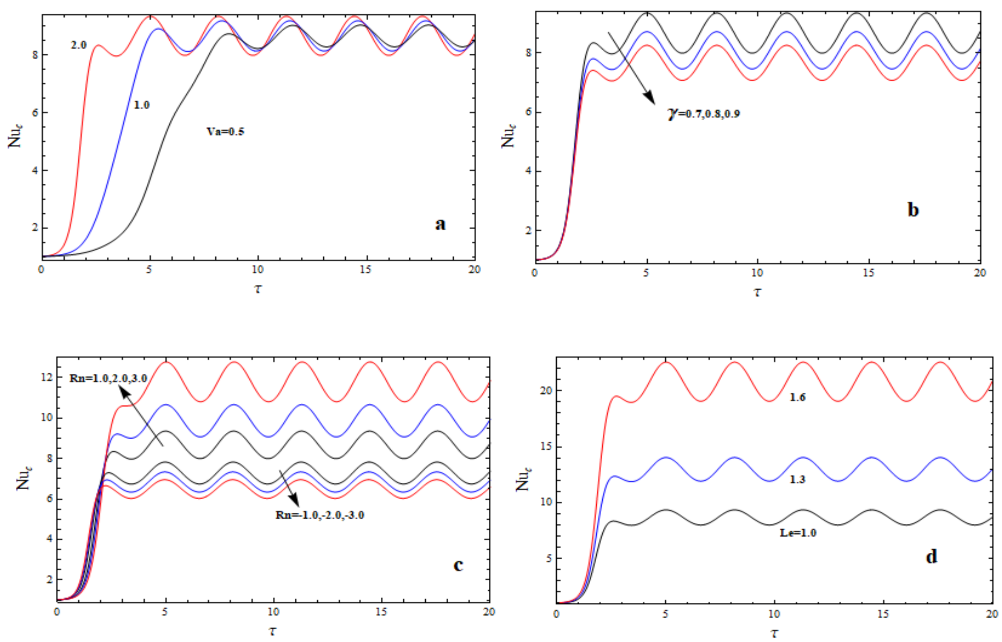

Fig. 5. Nano concentration transfer results for different system parameters (OPM case)

In the case of ordinary fluids these results are similar to Pr suggested by Bhadauria and Kiran [49-52]. The similar nature of the results of IPM are shown in Figures 4a and 5a for Va whereas its modulation effect varies.

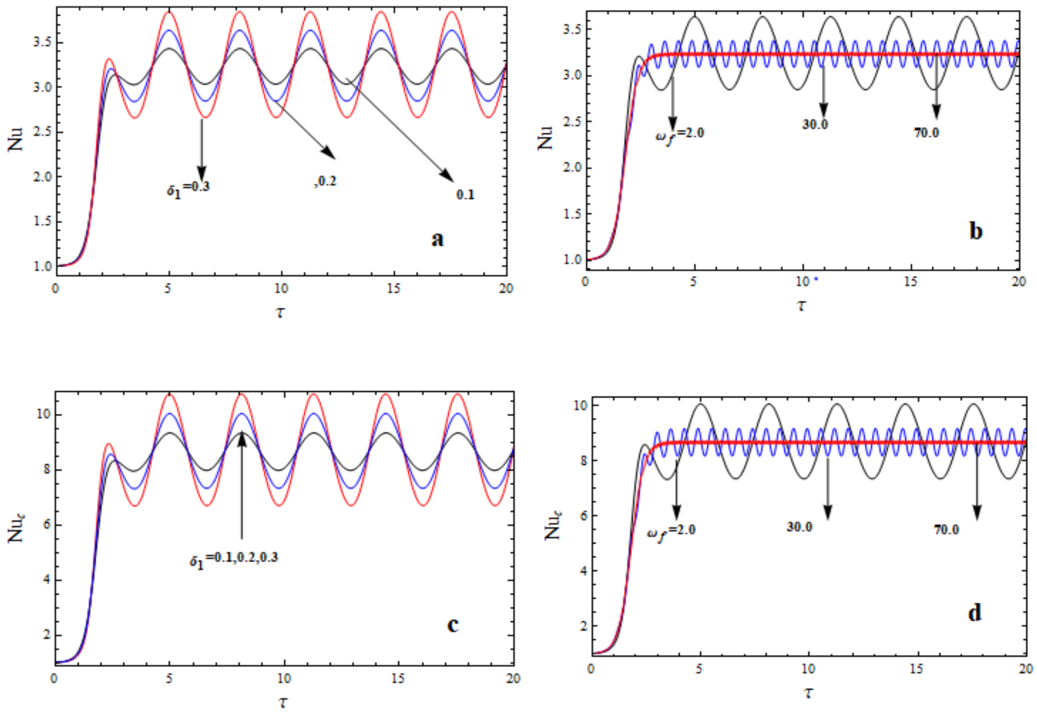

Fig. 6. Effect of amplitude and frequency of modulation on Nu and Nu_c

The heat capacity ratio γ was also taken as low values. Figures 2b and 3b show how Nu and Nu_c are affected by the heat capacity ratio γ, which reduces heat mass transfer in the layer for the IPM case. A similar pattern is seen for the OPM case shown in Figures 4b and 5b. The values of δ are taken between 0 and 1 due to pores volume fraction. Porosity parameter show its natural effect on Darcy convection. As porosity varies (i.e enhances) the flow rate increases through void spaces in the porous layer. The effect of porosity δ is not included in the figures due to same nature of results. The results may be compared with the results of Agarwal et al. [17, 18, 20, 53] and Kiran et al. [41, 47].

The concentration Rayleigh number Rn has the effect of enhancing the transport of heat and concentration, as shown in Figures 2c and 3c. The reader may note that the drop in heat and mass transfer is evident for negative values of Rn. As a result, Rn plays a dual role in transport media and can be used to regulate the transport of mass and heat. Most of the results related to Rn are followed by Agarwal et al. [17, 18, 20] and Kiran et al. [41-44]. Figures 4c and 5c show the OPM case results of a similar kind. Additionally, as previously shown by Bhadauria et al. [16] and Kiran et al. [40-44] that Le does not have a significant impact on Nu but for Nu_c. The graphical representation of Le is given in Figures 2d and 3d. On the other hand, Le has increasing influence on Nu_c than Nu. The OPM corresponding results are shown in Figures 4d and 5d show.

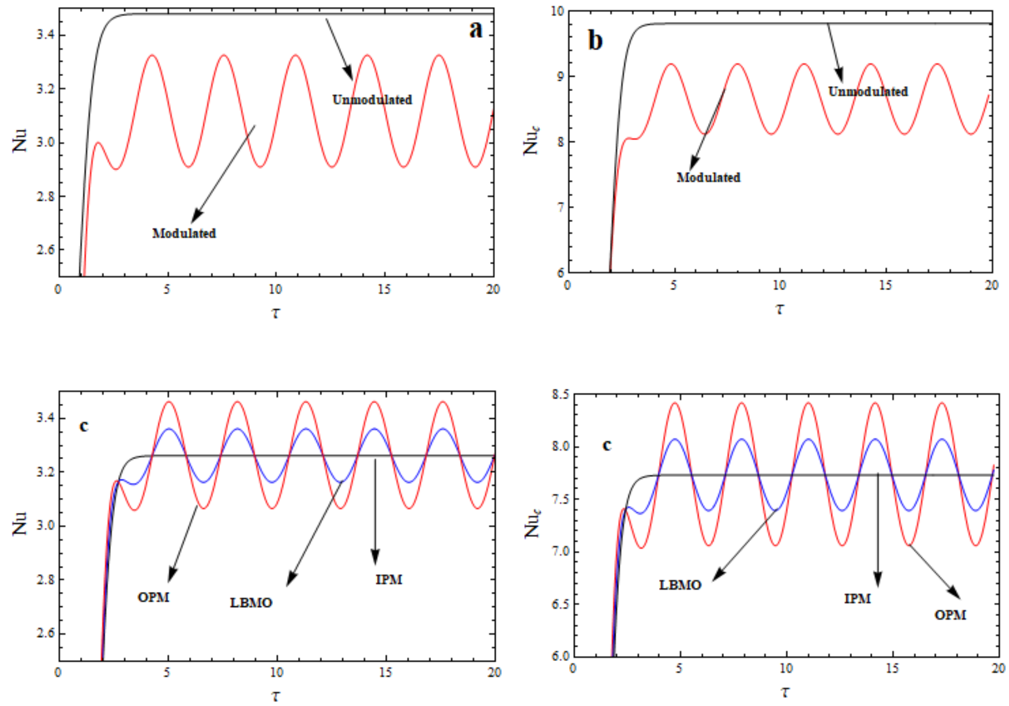

Fig. 7. Variation between madulated and un modulated system

The effect of temperature modulation (δ_1) on both heat and mass transfer is depicted in Figures 6a and 6c. The values of $\delta_1$are taken in the range of 0.1 to 0.5 and it is to promote the heat/mass transport. Similarly, the effect of frequency ω_f of modulation is depicted in the Figures 6b and 6d. It shows that a quite opposite results of δ_1 are obtained. Thus, heat mass transfer is improved by low modulation rates rather than high vibrational rates. It has been noticed that ω_f reduces heat/mass transport and are similar to Gresho et al. [32]. These results of nonofluids are compared with Bhadauria et al. [16] and Kiran et al. [40-42].

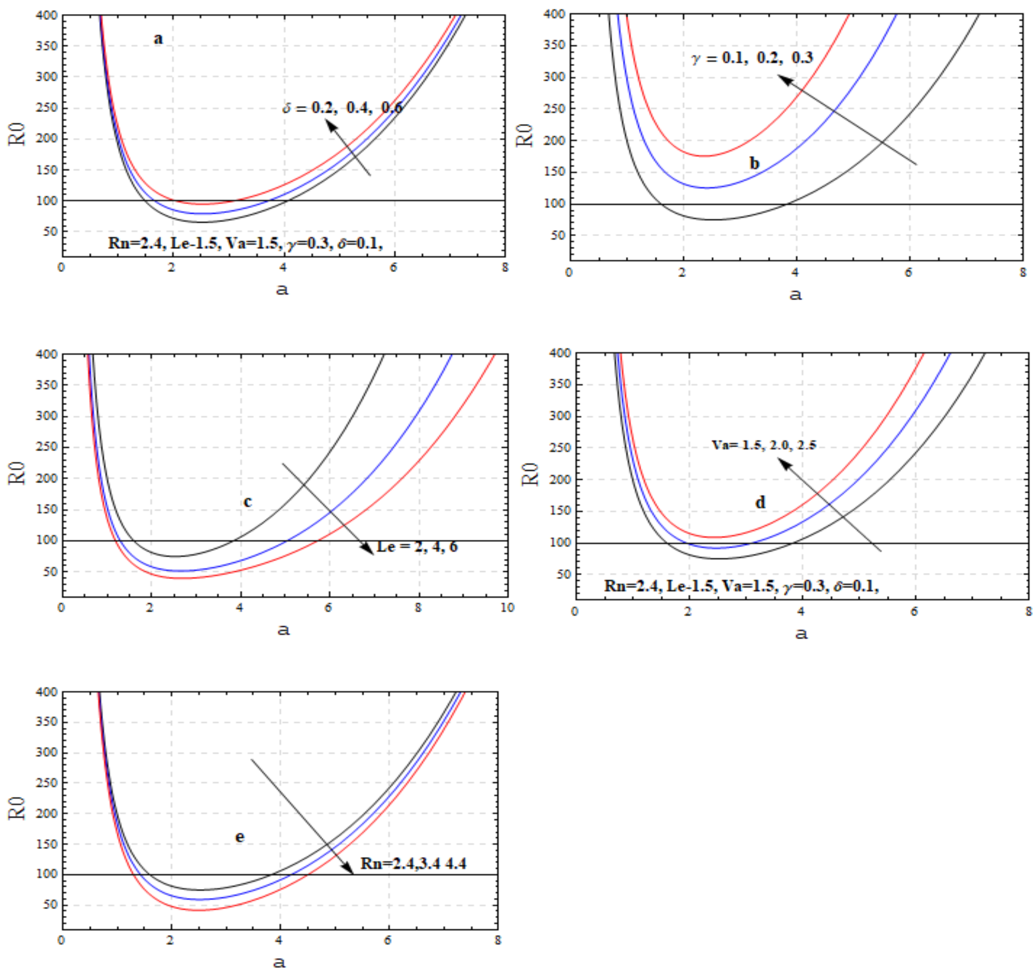

Fig. 8. Linear onsetstability curves for R0 versus wavenumber

Figures 7a and 7b provide a comparison of the modulated and unmodulated systems. The analytical solution for the convective amplitude of the unmodulated case is obtained while taking $\delta_1 = 0$. A modulated system promotes heat and mass transport more than a non-modulated system, as shown in Figures 7a and 7b. Figures 7c and 7d provide a comparison of temperature modulations. It is observed that

$$[Nu/Nu_c]_{OPM} > [Nu/Nu_c]_{LBM} > [Nu/Nu_c]_{IPM}$$

for any parameter values. It is obvious that one can regulate stability analysis by using three modulations. OPM and LBM are the two modulations that work best for controlling transport phenomena.

Figure 8 shows the onset stability curves. The values of critical R_0 and a for stationary and oscillatory modes are shown in Table 1. The results of Figure 8 are related to the table 1. It show that crucial R_0 increases upon the δ, γ and Va (see in Figures 8a,b,d) values and intensify the onset of convection. It is observed that the porosity parameter δ, heat capacity γ and Va increases the critical R_0 and delaying the onset of convection. However, the transport analysis is quite opposite of the onset convection because of the nonlinearity effects on Nu. Figures 8c and 8e show

how Le and Rn decrease R0 and delay the onset of convection and affect the enhancement in Nu and Nu_c. Our results are comparable to those of Bhadauria et al. [48] for two-dimensional convection rolls, but not for oscillatory case.

CONCLUSIONS

Weakly nonlinear nanofluid convection in a horizontal porous layer that is thermally modulated and heated and cooled from above has been studied. The top-heavy nanoparticle suspension has been taken into account in the presence of Brownian motion. On the basis of the previously provided results, the following observations have been made.

It is found that three boundary modulations have a considerable effect on controlling heat and mass transfer. The OPM and LBMO are effective at controlling the stability and transport phenomena. Out of the three modulations, OPM is the one that increases onset convection and improves heat and nano-concentration transport. The modulation effect has been described in terms of modulation amplitude and frequency. It is found that the modulation amplitude δ_1 increases Nu and Nuc for the intermediate values of time s. Also, Nu and Nuc are observed to decline as ω_f the modulation frequency increases. The concentration Rayleigh number Rn is found to have dual role on transport analysis. It is concluded that modulated nanolayers show better transfer results for oscillatory mode than stationary. Finally, stability curves (Figure 8) illustrate the characteristics of onset convection at various parameter values. From Table 1, it is found that Le, Va and Rn decrease R_{0c} values and advance onset convection, whereas δ and γ reduce R_{0c} values and delay onset convection.

Acknowledgment

The author expresses gratitude to the CBIT/VFSTR administration for providing the department’s research resources. They also expresses gratitude to Professor BS. Bhadauria and PG. Siddheshwar for their ideas towars the development of the research problems.

Declarations

Competing interests: The authors declare no competing interests.
Author contribution The author wrote the entire manuscript.

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