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Research Article

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Weak nonlinear analysis of nanofluid convection with g-jitter using the Ginzburg–Landau model

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Abstract: Nanofluid has emerged as a remarkable heat and mass transfer fluid due to its thermal characteristics. Despite this, continuing research is required to address problems in real applications and offer a solution for controlling transfer analysis. Therefore, in this study, the authors intend to model (Ginzburg–Landau equation) and analyze the two-dimensional nanofluid convection with gravity modulation. The perturbed analysis is adapted to convert the leading equations into Ginzburg–Landau equation. Lower amplitude (δ values from 0 to 0.5) values are taken since they influence transfer analysis. The values of Pr are considered as 0 to 2 to retain the local acceleration term in the system of equations. A lower amount of frequency of modulation (Ω values from 0 to 70) is sufficient to enhance the heat and mass transfer rates. It is found that g-jitter and concentration Rayleigh numbers control the stability of the system. The Prandtl number and the amplitude of modulation enhance nano-heat and nano-mass transfer. This shows a destabilizing effect of modulation on nano-convection. Also the nano-Rayleigh number Rn has a dual nature on the kinetic energy transfer for positive and negative signs. A comparison is made between modulated and unmodulated systems, and it is found that the modulated systems influences the stability problem than the unmodulated systems. Finally, it is found that g-jitter influences effectively to regulate the transport process in the layer.

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Nomenclature

Latin symbols		**Unit**
D_B	Brownian diffusion coefficient	m^2/s
$\vec{q}$	fluid velocity	m/s
d	dimensional layer depth	m
Rm	Basic density Rayleigh number, $\mathrm{Rm} = \frac{[\rho_p\phi_0 + \rho(1-\phi_0)]gd^3}{\mu\kappa_T}$	
Rn	concentration Rayleigh number, $\mathrm{Rn} = \frac{(\rho_p-\rho)(\phi_1-\phi_0)gd^3}{\mu\kappa_T}$	
Le	Lewis number, $\mathrm{Le} = \frac{\kappa_T}{D_B}$	
Pr	Prandtl number, $\mathrm{Pr} = \frac{\mu}{\rho\kappa_T}$	
(x^*, y^*, z^*)	Cartesian coordinates	m
N_B	modified particle-density increment, $N_B = \frac{(\rho c)_p}{(\rho c)_f}(\phi_1 - \phi_0)$	
$\vec{g}$	modulated gravity field	m/s^2
$\vec{q}$	nanofluid velocity	m/s
p	pressure	kg/(m s^2)
T	temperature	K
T_h	temperature at the lower wall	K
T_c	temperature at the upper wall	K
Ra	thermal Rayleigh number, $\mathrm{Ra} = \frac{\rho g\beta d^3(T_h - T_c)}{\mu\kappa_T}$	
τ	time	s
Greek symbols		
$(\rho c)_m$	Effective heat capacity of the porous medium	J/kgK

$(\rho c)_p$	effective heat capacity of the nanoparticle material	J/kgK
ρ_f	fluid density	kg/m^3
Ω	frequency of modulation	s^{-1}
$(\rho c)_f$	heat capacity of the fluid	J/kgK
k_c	horizontal wave number	
ρ_p	nanoparticle mass density	kg/m^3
β	proportionality factor	K^{-1}
μ	fluid viscosity	mPa s
κ_T	effective thermal diffusivity of the fluid	m^2/s
ν	kinematic viscosity μ/ρ_f	m^{4}Pa s/kg
ϕ	nanoparticle volume fraction	
ψ	stream function	m^2/s
δ	amplitude of modulation	m
Subscripts		
b	basic	
Superscripts		
*	dimensional variable	
′	perturbation variable	
Operators		
∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$	

1 Introduction

Convection in nanofluids is very important to analyze the thermal properties of nanoliquids. Studies related to nanofluids received a lot of interest from many authors due to their variety of behavior, and sudden enhancement in thermal conductivity. Due to the unexpected abnormal behavior of nanofluids, it is very important to investigate linear and nonlinear flow models. First, Choi [1] introduced the study of nanofluids, referring to fluids containing a scatter of particles (solid) whose characteristic dimension is 10 or 100 nm scaled. However, as compared to common fluids, some metals have a relatively high thermal conductivity. The fundamental idea behind these nanofluids was to suspend them in base fluids and give them thermal conductivity similar to metal. By adding the nanoparticles to base fluids, thermal conductivity may be enhanced by 15–40%. The majority of these heat-exchanging situations are found in modern engineering and research, including biomechanics, spinning machineries like nuclear reactors, food, geophysical problems, chemical processing, and the petroleum industry. A variety of nanofluid applications is due to their enhancing nature of heat and mass transfer with mixed nano-sized particles. Nanofluids control the transport processes that can be used in drug delivery systems.

Eastman *et al.* [2] reported that the abnormal enhancement in thermal conductivity of ethylene glycol increased by 40% for a nanofluid consisting of ethylene glycol containing approximately 0.3 volume percent Cu particle of diameter less than 10 nm. A sudden increment in critical heat flux due to the sudden progress of the thermal conductivity of nanoliquids was reported by Eastman *et al.* [3]. They reported that still there is the greatest challenge to overcome the potential class of heat transfer liquids. The incremental nature of the effective thermal conductivity is investigated experimentally by Masuda *et al.* [4]. Although the level of enhancement in liquids still has a question mark [2–5], the unique property of nanoliquids may suggest the use of nanoliquids in a variety of engineering applications, from advanced nuclear systems to drug delivery. Other relevant studies of nanofluids and their convection instabilities are given by refs. [6–10]. The studies of nanoliquids in detail are presented in the book of Nield and Kuznetsov [11].

Rayleigh Benard convection (RBC) of nanofluids has been investigated for the past few decades. Here, Buongiorno and Hu [12] and Kuznetsov and Nield [13] studied nanoliquids for different physical configurations. The dispersion of nano-liquid particles may acquire diminish or progress the onset convection and thereby heat and nano-mass transport results. It is a because of the presence of concentration gradients of the nanoparticles. Most of the research indicates that the instability of any flow is due to the buoyancy force effect. This force is accompanied by the conservation of nanoliquids and does not depend on the nature of Brownian motion and thermophoresis. Thus, it is needed to alternate the gravity field along with nanofluids to control instability in the media. In fact, the Brownian motion produces drastic nature of enhancement only when temperature and particle flow are together.

Chamkha *et al.* [14] investigated the boundary-layer flow with heat and mass transfer of a nanofluid along a horizontal stretching plate in the presence of a transverse magnetic field, melting, and heat generation, or absorption effects. The numerical modeling of a steady, laminar natural convection flow in a triangular enclosure partially heated from below and with a cold inclined wall was studied by Ahmed *et al.* [15]. A steady of magnetohydrodynamics (MHD) natural convection subject to varied boundary conditions at the sidewalls in the presence of the inclined magnetic field was reported by Mansour *et al.* [16]. The same problem has been extended to the porous layer by Rashad *et al.* [17], and the results of velocity profiles and heat and mass transfer were drawn. Mourad *et al.* [18] examined the MHD-free convection in a fined cold wavy-walled porous enclosure with a hot elliptic inner cylinder occupied by hybrid Fe_3O_4-MWCNT/water nanofluid.

It is important to maintain the rate of transport process in the media by applying modulations like temperature or gravity. These are important in real day-to-day applications and various engineering problems. Sinusoidal variations of plate modulation is known as thermal modulation, and it is introduced by Venezian [19] for the linear mode. The instability of the linear mode is known as the onset of convection, which gives just a critical state of the Rayleigh number. The effect of gravity modulation, *i.e.*, vertical vibrations, on RBC was first reported in ref. [20]. Some related works on g-jitter in recent times were given by Malashetty and Basavaraj [21], Shu *et al.* [22], Rogers *et al.* [23], Boulal *et al.* [24], and Umavathi [25] are few. Their research focused on external constraints to regulate convective heat and mass flow models. Umavathi [25] reported the temperature modulation effect on nanofluid Darcy convection by performing a linear stability analysis. It was reported that thermal modulation may be used to regulate the onset of nanofluid convection. There were no study about g-jitter on the RBC of nanofluids with Ginzburg–Landau (GL) model. In this article, the g-jitter effect of RBC has been studied by performing nonlinear analyses. Consequently, the thermal and concentration Nusselt numbers are calculated as a function of other physical parameters.

The study of nonlinear finite amplitude convection in nanofluids was reported in the studies by Bhadauria *et al.* [26], Agarwal *et al.* [27,28]. The rotational effect is accounted for nanofluid convection in the rotating porous medium. They have used a truncated representation of the Fourier series, to convert nonlinear partial differential equation (PDE) into a set of simultaneous ordinary differential equations. Their study reported that nanofluids are modeled to enhance thermal and concentration transport. The other studies that are similar to refs [25–28] were given in refs [29–31], in which binary nano-convection was investigated. In all the aforementioned studies, the results are reported on onset convection and transport phenomenon without modulation and different thermal boundary conditions. Agarwal and Bhadauria [32] investigated nano-liquid convection in the presence of local thermal non-equilibrium (LTNE). It is observed that LTNE can be used to alternate heat and mass transfer in the system.

There were no data about the studies of nanofluids for a nonlinear mode of convection under modulation. Bhadauria and Kiran introduced the modulation effect on nanofluid convection for nonlinear modes. The g-jitter effect on nanoconvection was given by Bhadauria and Kiran [33,34]. Modulation regulates the transport phenomenon with the help of a finite amplitude. Kiran [35] studied the nano-nonlinear instability in a viscoelastic porous medium (saturated by nanofluid) under gravitational modulation was given by Kiran [35]. The same problem for internal heating was presented in Kiran *et al.* [36]. The effect of out of phase modulation and lower boundary modulation on nano-convection was introduced by Kiran and Narasimhulu [37,38]. Here, they have found that the modulation effect not only controls the transport phenomena but also chaotic convection. The effect of throughflow on nanofluid convection was given by Kiran *et al.* [39]. It was found that throughflow shows both inflow and outflow enhance or diminish energy transfer in media. The recent studies of g-jitter on RBC and Darcy convection are given by Kiran [40–42]. In the literature, till date, no data were reported on the GL model for nonlinear nanofluid convection. The GL model is used to find the finite amplitude of nonlinear thermal instability.

In this study, the effect of gravity modulation for the classical Rayleigh–Benard problem of nanofluids is investigated. The GL model is applied to determine the convective amplitude. The solution of the GL equation is obtained based on the Cartesian coordinate procedure. The modulation of the fluid flow on the free boundary conditions is calculated and represented graphically for various relevant parameters Kiran *et al.* [35,36].

2 Problem formulation

A horizontal layer of nanofluid is considered between two plates at $z = 0$ (lower plate) and $z = d$ (upper plate). This configuration is heated from beneath and cooled the upper one. The boundary of the plates is taken to be impermeable and perfectly conducting. It is infinitely extended in horizontal, y-direction, and z-axis vertically upward (Figure 1). Here, T_h is the temperature at the lower plate and T_c is the temperature at the upper plate. Detailed mathematical equations are presented in refs [6,7,43,44].

$$\nabla\cdot \vec{q} = 0, \tag{1}$$

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + \vec{q}\cdot\nabla\vec{q}\right) = -\nabla p + \mu\nabla^2\vec{q} + [\phi\rho_p + (1-\phi)\{\rho_f(1-\beta(T-T_c))\}]\vec{g}, \tag{2}$$

$$(\rho c)_f\left[\frac{\partial T}{\partial t} + \vec{q}\cdot\nabla T\right] = k_f\nabla^2 T + (\rho c)_p D_{\mathrm{B}}\nabla\phi\cdot\nabla T, \tag{3}$$

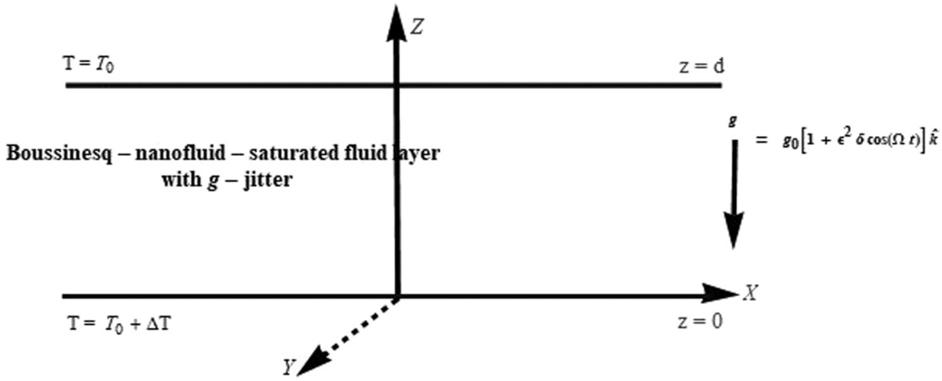

Figure 1: Physical configuration of gravity modulation.

$$\frac{\partial \phi}{\partial t} + \vec{q} \cdot \nabla \phi = D_{\mathrm{B}} \nabla^2 \phi, \tag{4}$$

$$\vec{g} = g(1 + \varepsilon^2 \delta \cos(\Omega t)), \tag{5}$$

where $\vec{q} = (u, v, w)$ is the velocity of the fluid. The other related parameters are given by fluid density ρ, effective heat capacities $(\rho c)_f$ and $(\rho c)_p$, and k_f thermal conductivity. Also, δ and Ω are the amplitude and frequency of g-jitter, respectively. Here, D_{B} is the Brownian diffusion coefficient arises due to nanofluid motions. The following stress-free boundary conditions for T and ϕ and velocity are taken.

$$\vec{q} = 0, \quad T = T_h, \quad \phi = \phi_0 \quad \text{at } z = 0, \tag{6}$$

$$\vec{q} = 0, \quad T = T_c, \quad \phi = \phi_1 \quad \text{at } z = d. \tag{7}$$

For dimensionless analysis, the physical variables are taken as follows: $(x^*, y^*, z^*) = (x, y, z)d^{-1}$, $t^* = \frac{t\alpha_f}{d^2}$, $T^* = \frac{T - T_c}{T_h - T_c}$, $(u^*, v^*, w^*) = \frac{(u, v, w)d}{\alpha_f}$, $p^* = \frac{pd^2}{\mu \alpha_f}$, $\phi^* = \frac{\phi - \phi_0}{\phi_1 - \phi_0}$, and $\alpha_f = \frac{k_f}{(\rho c)_f}$. By using the transformations into Eqs. (1)–(7), we the following system of equations is arrived, that define dimensionless PDE (after dropping the asterisk):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{8}$$

$$\begin{aligned} &\frac{1}{\mathrm{Pr}} \frac{\partial \vec{q}}{\partial t} + \mathrm{Pr}^{-1}(\vec{q} \cdot \nabla \vec{q}) - \nabla^2 \vec{q} \\ &= -\nabla p + (\mathrm{Ra}_T T + \mathrm{Rn}\phi - \mathrm{Rm})(1 + \varepsilon^2 \delta \cos(\Omega t))\vec{k}, \end{aligned} \tag{9}$$

$$\frac{\partial T}{\partial t} + \vec{q} \cdot \nabla T = \nabla^2 T + \frac{N_{\mathrm{B}}}{\mathrm{Le}} \nabla \phi \cdot \nabla T, \tag{10}$$

$$\frac{\partial \phi}{\partial t} + \vec{q} \cdot \nabla \phi = \frac{1}{\mathrm{Le}} \nabla^2 \phi, \tag{11}$$

$$(\vec{q}, T, \phi) = (0, 1, 0) \quad \text{at } z = 0, \tag{12}$$

$$(\vec{q}, T, \phi) = (0, 0, 1) \quad \text{at } z = 1. \tag{13}$$

The dimensionless parameters are given by thermal Rayleigh–Darcy number, $\mathrm{Ra}_T = \frac{\rho g \beta d^3 (T_h - T_c)}{\mu k_T}$, basic density Rayleigh number, $\mathrm{Rm} = \frac{[\rho_p \phi_0 + \rho(1 - \phi_0)]gd^3}{\mu k_T}$, concentration Rayleigh number, $\mathrm{Rn} = \frac{(\rho_p - \rho)(\phi_1 - \phi_0)gd^3}{\mu k_T}$, Lewis number, $\mathrm{Le} = \frac{k_T}{D_{\mathrm{B}}}$, Prandtl number, $\mathrm{Pr} = \frac{\mu}{\rho_f k_T}$ and modified particle–density increment, $N_{\mathrm{B}} = \frac{(\rho c)_p (\phi_1 - \phi_0)}{(\rho c)_f}$.

At the initial state of convection, the physical variables are in the conduction mode in the z-direction and are given by

$$\vec{q} = 0, \quad p = p_b(z, t), \quad T = T_b(z), \quad \phi = \phi_b(z). \tag{14}$$

The following is obtained while substituting equation (14) in Eqs (10) and (11):

$$\frac{\mathrm{d}^2 T_b}{\mathrm{d}z^2} + \left(\frac{N_{\mathrm{B}}}{\mathrm{Le}}\right)\left(\frac{\mathrm{d}\phi_b}{\mathrm{d}z}\right)\left(\frac{\mathrm{d}T_b}{\mathrm{d}z}\right) = 0. \tag{15}$$

The second term in the aforementioned equation is very small. According to the studies by Kuznetsov and Nield [13] and Agarwal and Bhadauria [45], applying the order analysis, the following equations may be obtained from Eqs. (11) and (15):

$$\frac{\mathrm{d}^2 T_b}{\mathrm{d}z^2} = 0, \quad \frac{\mathrm{d}^2 \phi_b}{\mathrm{d}z^2} = 0. \tag{16}$$

To solve Eq. (16), we use boundary conditions from Eqs. (12) and (13) as follows:

$$T_{\text{basic}} = 1, \quad \phi_{\text{basic}} = 0 \quad \text{at } z = 0, \tag{17}$$

$$T_{\text{basic}} = 0, \quad \phi_{\text{basic}} = 1 \quad \text{at } z = 1. \tag{18}$$

Equation (16) solves basic T and ϕ using Eqs (17) and (18):

$$T_{\text{basic}} = 1 - z, \tag{19}$$

$$\phi_{\text{basic}} = z. \tag{20}$$

After the completion of the basic state, the system enters to convection mode. At this state, the basic state, imposed by small perturbed quantities:

$$\vec{q} = \vec{q}', \quad p = p_b + p', \quad T = T_b + T', \quad \phi = \phi_b + \phi'. \tag{21}$$

Inserting Eq. (21) into Eqs (8) and (11), and using Eqs (19) and (20), and after omitting the pressure term p, the following system is obtained for a two-dimensional flow:

$$-\nabla^4\psi + \frac{1}{\Pr}\frac{\partial}{\partial t}(\nabla^2\psi) = \frac{\partial(\psi, \nabla^2\psi)}{\partial(x,z)} - g_m\mathrm{Ra}_T\frac{\partial T}{\partial x} - g_m\mathrm{Rn}\frac{\partial\phi}{\partial x}, \tag{22}$$

$$\frac{\partial\psi}{\partial x} - \nabla^2 T = -\frac{\partial T}{\partial t} + \partial(\psi, T)/(\partial(x,z)), \tag{23}$$

$$-\frac{\partial\psi}{\partial x} = \frac{1}{\mathrm{Le}}\nabla^2\phi - \frac{\partial\phi}{\partial t} + \partial(\psi,\phi)/(\partial(x,z)), \tag{24}$$

where $g_m = (1 + \varepsilon^2\delta\cos(\Omega t))$. Since we assume low variations, we consider $\tau = \varepsilon^2 t$. Equations (22)–(24) are solved in subject to the following conditions:

$$\left(\psi, \frac{\partial^2\psi}{\partial z^2}, T, \phi\right) = (0,0,0,0) \quad \text{at } z = 0, 1. \tag{25}$$

3 Convective nanofluid amplitude

Introducing the following asymptotic expansions in Eqs (22)–(24) [46]:

$$\mathrm{Ra}_T = R_{0c} + \varepsilon^2 R_2 + \varepsilon^4 R_4 + \ldots,$$

$$(\psi, T, \phi) = \varepsilon(\psi,T,\phi)_1 + \varepsilon^2(\psi,T,\phi)_2 + \varepsilon^3(\psi,T,\phi)_3 + \ldots,$$

where R_{0c} represents the critical Rayleigh number in the absence of modulation. The system is solved for various powers of ε. At the lowest order. *i.e.*, ε, the following is obtained:

$$M\begin{bmatrix}\psi_1\\ T_1\\ \phi_1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}, \tag{26}$$

where $M = \begin{bmatrix} -\nabla^4 & R_{0c}\frac{\partial}{\partial x} & -\mathrm{Rn}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial x} & -\nabla^2 & 0\\ -\frac{\partial}{\partial x} & 0 & -\frac{1}{\mathrm{Le}}\nabla^2\end{bmatrix}$.

The solutions of the above system are taken as (to satisfy Eq. (25))

$$\psi_1 = \mathbb{B}(\tau)\sin(k_c x)\sin(\pi z), \tag{27}$$

$$T_1 = -\frac{k_c}{\delta^2}\mathbb{B}(\tau)\cos(k_c x)\sin(\pi z), \tag{28}$$

$$\phi_1 = \frac{k_c}{\delta^2}\mathrm{Le}\mathbb{B}(\tau)\cos(k_c x)\sin(\pi z), \tag{29}$$

where $\delta^2 = k_c^2 + \pi^2$. The critical Rayleigh number and the corresponding wavenumber are given by

$$R_{0c} = \frac{\delta^6}{k_c^2} - \mathrm{RnLe}, \quad k_c = \frac{\pi}{\sqrt{2}},$$

and these results are given by Venezian [19], Rana and Agarwal [30] and Agarwal and Bhadauria [32]. In the case of the second-order system, *i.e.*, the ε^2 order of the following system is obtained:

$$M\begin{bmatrix}\psi_2\\ T_2\\ \phi_2\end{bmatrix} = \begin{bmatrix}R_{21}\\ R_{22}\\ R_{23}\end{bmatrix}, \tag{30}$$

$$R_{21} = 0, \tag{31}$$

$$R_{22} = \frac{\partial\psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial\psi_1}{\partial z}\frac{\partial T_1}{\partial x}, \tag{32}$$

$$R_{23} = \frac{\partial\psi_1}{\partial x}\frac{\partial\phi_1}{\partial z} - \frac{\partial\psi_1}{\partial z}\frac{\partial\phi_1}{\partial x}. \tag{33}$$

The following second-order solutions are obtained with the help of the first-order solutions given in Eq. (29):

$$\psi_2 = 0, \tag{34}$$

$$T_2 = -\frac{k_c^2}{8\pi\delta^2}\mathbb{B}^2(\tau)\sin(2\pi z), \tag{35}$$

$$\phi_2 = \frac{k_c^2}{8\pi\delta^2}\mathrm{Le}\mathbb{B}^2(\tau)\sin(2\pi z). \tag{36}$$

The energy transfer coefficient of the model is given by $\mathrm{Nu}(\tau)$:

$$\mathrm{Nu}(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}\left(\frac{\partial T_2}{\partial z}\right)\mathrm{d}x\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}\left(\frac{\partial T_b}{\partial z}\right)\mathrm{d}x\right]_{z=0}}, \tag{37}$$

$$\mathrm{Nu}(\tau) = 1 + \frac{k_c^2}{4\delta^2}\mathbb{B}^2(\tau). \tag{38}$$

The nano-Nusselt number $\mathrm{Nu}_\phi(\tau)$ is defined as same as the Nusselt number, and it is given by

$$\mathrm{Nu}_{\phi}(\tau) = 1 + \frac{k_c^2}{4\delta^2}\mathrm{Le}\mathbb{B}^2(\tau). \tag{39}$$

The following system is obtained in third order ε^3:

$$M\begin{bmatrix}\psi_3\\ T_3\\ \phi_3\end{bmatrix} = \begin{bmatrix}R_{31}\\ R_{32}\\ R_{33}\end{bmatrix}. \tag{40}$$

The terms of Eq. (40) are given by

$$R_{31} = -\frac{1}{\mathrm{Pr}}\frac{\partial}{\partial\tau}(\nabla^2\psi_1) - R_2\frac{\partial T_1}{\partial x} - R_{0c}\delta\cos(\Omega\tau)\frac{\partial T_1}{\partial x} + \mathrm{Rn}\delta\cos(\Omega\tau)\frac{\partial\phi_1}{\partial x}, \tag{41}$$

$$R_{32} = -\frac{\partial T_1}{\partial\tau} + \frac{\partial\psi_1}{\partial x}\frac{\partial T_2}{\partial z}, \tag{42}$$

$$R_{33} = -\frac{\partial\phi_1}{\partial\tau} + \frac{\partial\psi_1}{\partial x}\frac{\partial\phi_2}{\partial z}. \tag{43}$$

By using the expressions of ψ_1, T_1, and T_2 into Eqs (41)–(43), the expressions for R_{31}, R_{32}, and R_{33} are obtained. The following the GL equation is obtained with the help of the solvability condition [40–42].

$$A_1\frac{\mathrm{d}\mathbb{B}}{\mathrm{d}\tau} = A_2\mathbb{B}(\tau) - A_3\mathbb{B}(\tau)^3, \tag{44}$$

where $A_1 = \frac{\delta^2}{\mathrm{Pr}} + \frac{k_c^2}{\delta^4}(R_{0c}) + \frac{\mathrm{Rn}k_c^2\mathrm{Le}}{\delta^4}(\mathrm{Le})$, $A_2 = \left[\frac{R_2k_c^2}{\delta^2} + \left(\frac{R_{0c}k_c^2}{\delta^2} + \frac{\mathrm{Rn}k_c^2}{\delta^2}(\mathrm{Le})\right)\delta\cos(\omega\tau)\right]$, and $A_3 = \frac{k_c^4}{8\delta^4}(R_{0c} - \mathrm{RnLe}^3)$.

The GL equation (44) is a Bernoulli differential equation with time-dependent coefficients. In general, obtaining its solution (in terms of $\mathbb{B}(\tau)$) is difficult. Thus, it is evaluated numerically with Mathematica, subject to an initial condition $\mathbb{B}(0) = b_0$, where b_0 is the initial amplitude. $R_2 = R_{0c}$ is considered for local nonlinearity.

4 Direct solutions

Equation (45) is an analytical solution of $\mathbb{B}(\tau)$ for the unmodulated case ($\delta = 0$):

$$\mathbb{B}_u(\tau) = \frac{1}{\sqrt{\left(\frac{A_3}{2A_2} + C_1\exp\left(\frac{-2A_2}{A_1}\right)\right)}}, \tag{45}$$

where C_1 is a constant. Thermal and nanoparticle concentration coefficients (Nu and Nu_{ϕ}) are calculated from Eqs (38) and (39), respectively.

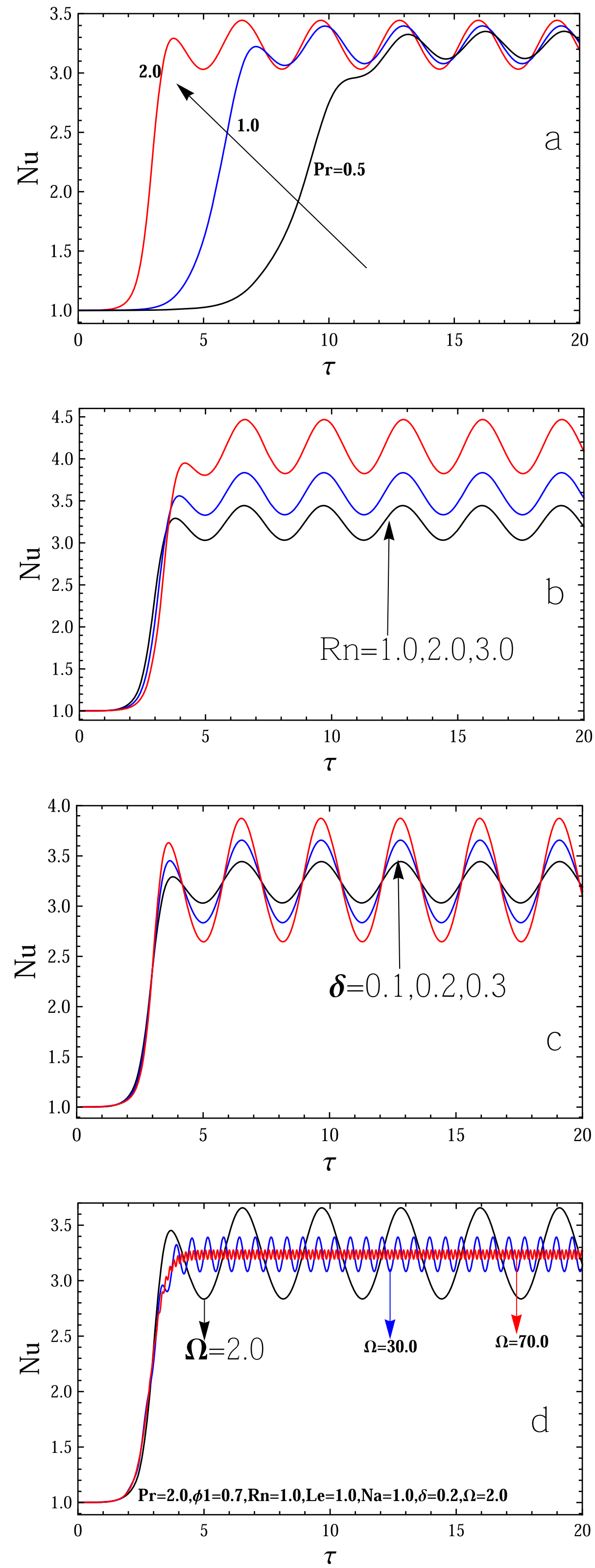

Figure 2: Heat transfer results based on the effect of (a) Va, (b) Rn, (c) δ, and (d) Ω.

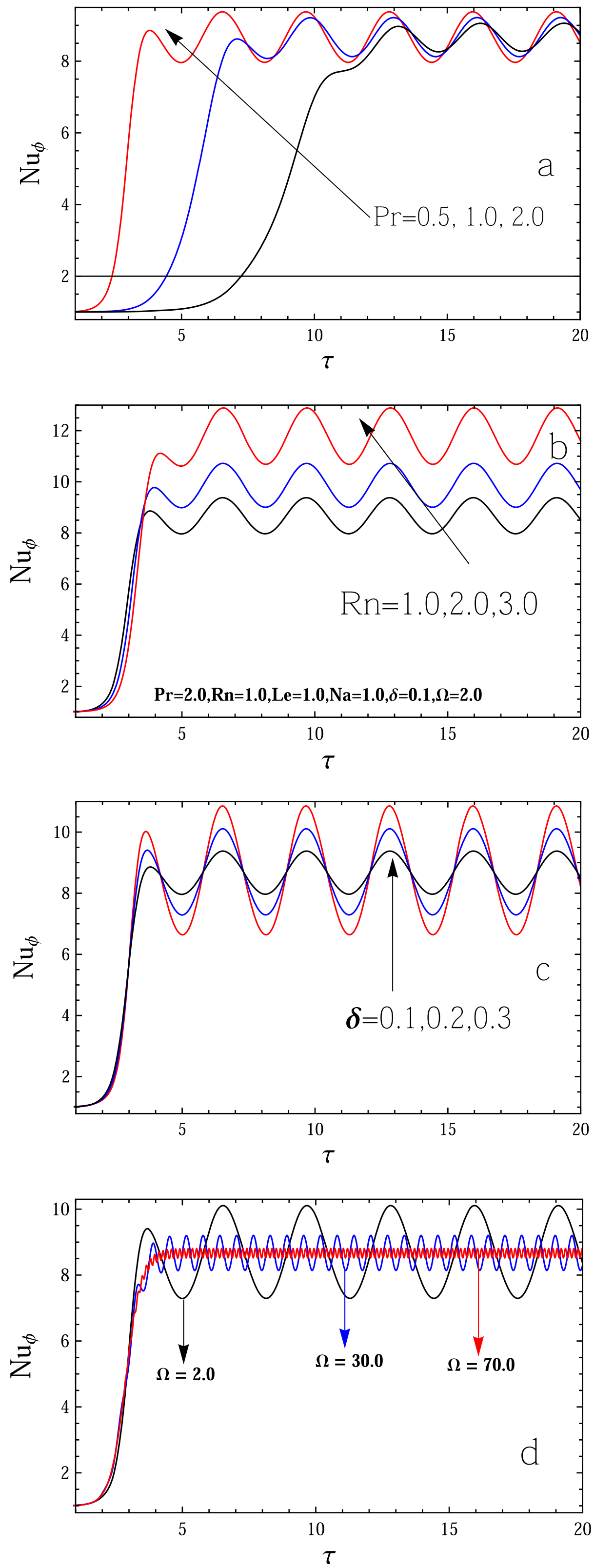

Figure 3: Nanofluid mass transfer results based on the effect of (a) Va, (b) Rn, (c) δ, and (d) Ω.

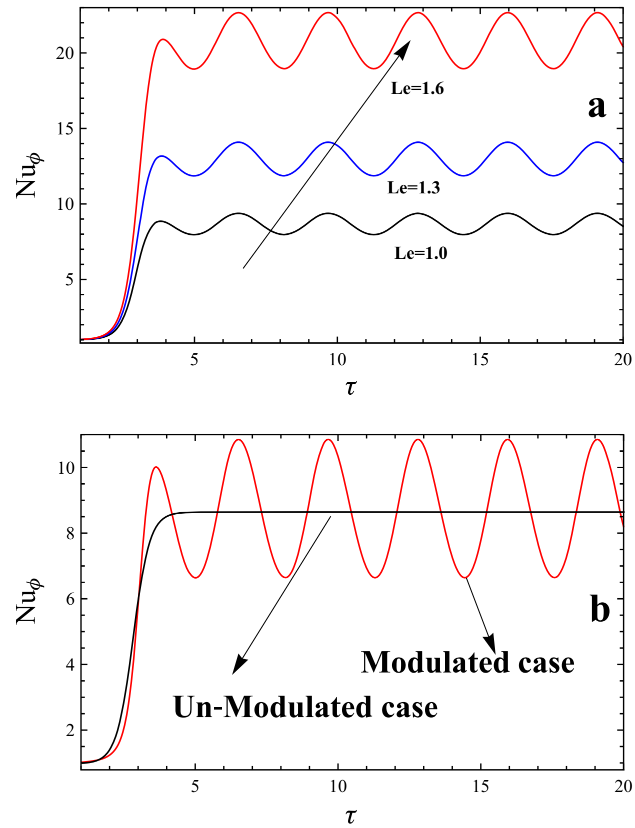

Figure 4: Heat transfer results based on the effect of (a) Le and (b) comparison.

5 Results and discussion

The gravity modulation effect on Rayleigh–Benard nanoconvection is investigated by performing a weakly nonlinear stability analysis. The gravity modulation is of $O(\varepsilon^2)$ order and determines the slow variation of convection. This assumption leads to obtain a finite convective amplitude of nanoconvection. The present model is better than the Lorenz model to derive the amplitude equation. In fact, this nonlinear study provides a detailed investigation of heat/mass transport. In general, it is not possible for a linear theory to study transport phenomena. However, external regulations like gravity, thermal, and rotation, are important to study thermal instability and control instability. Therefore, in this article, the g-jitter effect is being taken to examine the fluctuation of transport of heat/mass transfer. Recently, and a decade before, only linear models were developed with finite terms of the Fourier series. So far, no GL model is applied for nanofluid

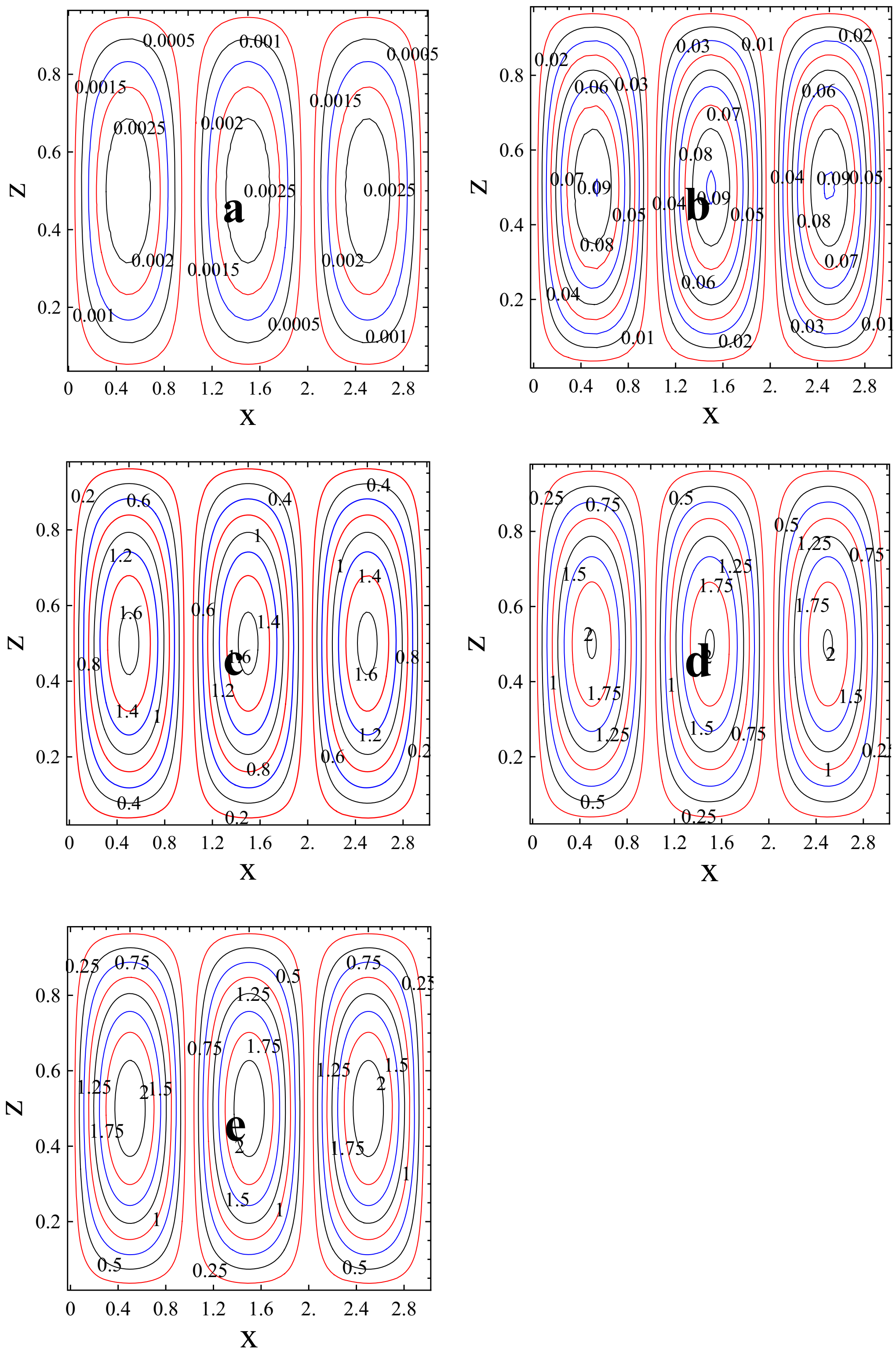

Figure 5: Streamlines at (a) $\tau = 0.0$, (b) $\tau = 1.0$, (c) $\tau = 3.0$, (d) $\tau = 5.0$, and (e) $\tau = 7.0$.

convection in the case of nonlinear thermal instabilty. Hence, the GL model is employed for a weakly nonlinear nano-convection with modulation.

The effect of g-jitter on heat/mass transport is reported in Figures 2–6. In our investigation, the variables Pr, Rn, Le, δ, and Ω are the physical variables occurred to

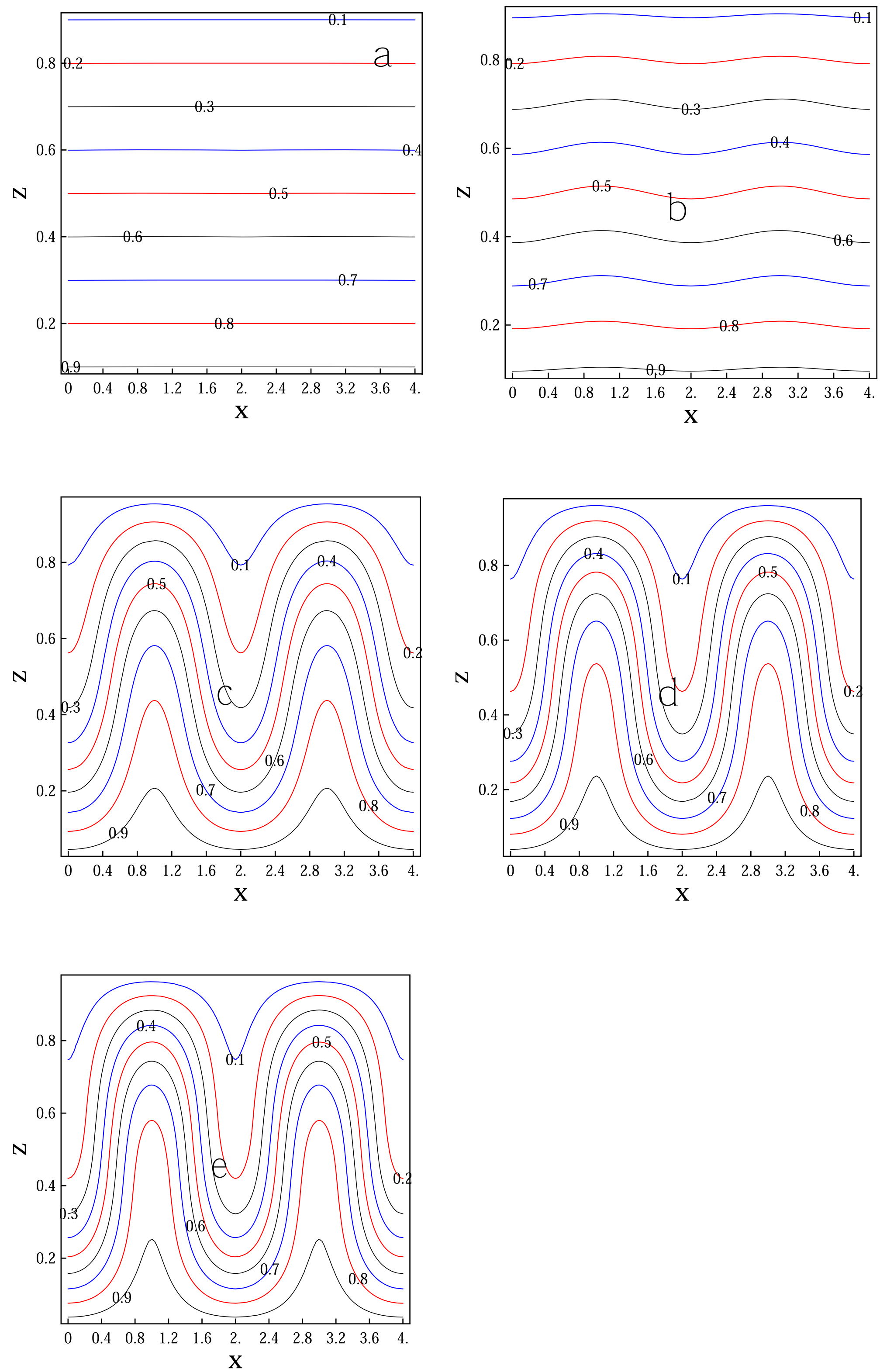

Figure 6: Isotherms at (a) $\tau = 0.0$, (b) $\tau = 1.0$, (c) $\tau = 3.0$, (d) $\tau = 5.0$, and (e) $\tau = 7.0$.

discuss convective transport. The first three parameters are related to the fluid layer and the next two parameters concern the external mechanism of controlling convection. The fluid layer is not considered to be highly viscous; and therefore, only moderate values of Pr are taken for calculations. Because of small amplitude modulation, the values of δ are considered to be small.

A weakly nonlinear thermal instability is performed for the nanoconvection using the GL model. The transfer coefficients of heat/mass transfer are given in terms of the Nusselt (Nu) and the concentration Nusselt numbers $Nu_\phi(\tau)$. These parameters are calculated as a function of the above-mentioned physical variables. The motivation of the present article is taken from the studies by Kiran *et al.* [35–42]. Here, the authors considered the g-jitter effect of nano-RBC convection with free thermal boundaries. So far, no one has considered this nonlinear nanofluid convection with the GL model. The results of our study are discussed in the following paragraphs.

The Prandtl number Pr is to increase the heat and concentration transfer for low values of time, and further increment in time the similar effect can be observed in Figures 2(a) and 3(a). The effect of Pr is quite natural to enhance transport phenomenon. For nonoliquids, one may observe the related studies of Pr followed by Malashetty and Basavaraj [21], Agarwal and Bhadauria [29,31], and Bhadauria *et al.* [34]. The influence of concentration Rayleigh number Rn is to increase the heat and concentration transfer in the layer (Figures 2(b) and 3(b)). Here, the reader may note that the negative values of Rn show reduction in heat mass transfer. The negative values of Rn represents shrink in the particles. Due to the reputation of figures, Rn figure is not inserted. The Rn has a dual role in transport media, which is used to control energy and mass transfer process. Most of the results of Rn are presented in the studies by Agarwal *et al.* [28–31] and Kiran and Manjula [47].

Figures 2(c) and 3(c) show the impact modulation amplitude δ on both heat and mass transfer. The values of δ are considered in the range from 0.1–0.5 to enhance the heat and mass transfer. Similarly, the effect of frequency Ω of modulation is shown in Figures 2(d) and 3(d). It shows that quite opposite results of δ are obtained. This means that at low modulation rates, *i.e.*, low-frequency cases $\Omega = 2$, the heat and mass transfer are more than that at high vibrational rates $\Omega = 70$. It is noted that the frequency of modulation Ω reduces heat/mass transfer and concludes the results reported by Gresho and Sani [20]. The reader may notice that the low-frequency g-jitter needs to be considered to maximize the transport process. Our present results of nanofluids can be compared with the studies by Kiran *et al.* [35–39] and Bhadauria and Agarwal [26].

It is found that Le does not have an effect on heat transfer and confirms the studies of Bhadauria and Agarwal [26] and Kiran *et al.* [35–39]. It is clear that Le is related to particle concentration by its definition. Its figure is not included to avoid graphical representation. Le enhances only the concentration transport, and it is depicted in Figure 4a. Eq. (45) gives an analytical amplitude of convection for unmodulated case. By using this amplitude, a comparison between the modulated system and the unmodulated systems is presented in Figure 4b. It shows that there is a sudden increment in $Nu(\tau)$ and $Nu_\phi(\tau)$ for low instance of time τ and becomes steady for higher values of time τ. But, in case of the modulated system, it shows oscillatory behavior for both $Nu(\tau)$ and $Nu_\phi(\tau)$. Here, it is noticed that only results of $Nu_\phi(\tau)$ are presented.

In Figures 5 and 6, streamlines and isotherms are drawn for fixed values of the system parameters $\tau = 0.0, 0.1, 0.5, 1.0,$ and 2.0 for $Pr = 1.0$, $Rn = 10.0$, $\delta = 0.3$, $Le = 1.0$, and $\Omega = 2.0$, respectively. The natural convection nature is observed in Figures 5 and 6. Initially, *i.e.*, at the conduction state the fluid is at the rest mode, and no difference is observed in Figures 5, 6(a) and (b). Both streamlines and isotherms are in normal state. However, Figure 5(c) show that the magnitude of streamlines increases upon increasing the values of slow times. A similar nature is observed for isotherms which are given in Figure 6(c). Further, Figures 5(c) and 6(c) loses their evenness and show that the convection is in progress. The convection becomes faster by further increasing the value of time τ. At the later point of time, the system achieves its steady state beyond ($\tau = 5.0$) as there is no change in the streamlines and isotherms (Figures 5(d, e) and 6(d, e)).

6 Conclusion

Weakly nonlinear nanofluid convection in a horizontal fluid layer that is gravity modulated and heated and cooled from above has been studied. The top-heavy nanoparticle suspension has been taken into account in the presence of Brownian motion. On the basis of the previously provided results, the following observations have been made.

It is found that gravity modulation has a considerable effect on controlling heat and mass transfer. The numbers Nu and Nu_ϕ are calculated to study heat and concentration transports in the layer with the help of the GL equation. The modulation effect has been described in terms of modulation amplitude and frequency. The fluid property parameter Pr is to enhance the heat $Nu(\tau)$ and mass $Nu_\phi(\tau)$ transfer. The Lewis number Le shows the growth rate on $Nu_\phi(\tau)$ as it is related to the ratio of nano-particle heat transfer to that of convective mass transfer. It is found that the modulation amplitude δ increases Nu and Nu_ϕ for the intermediate values of time (τ). Also, Nu and $Nu_\phi(\tau)$ are observed to decline as Ω modulation frequency increases. The concentration Rayleigh number Rn is found to have a dual role in the transport analysis. It is found that modulated nanolayers enhance transport analysis than stationary models. Furthermore, it is concluded that the suitable adjustment of the values of δ, Ω, and Rn may have control over transfer analysis.

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