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Generation of fractals via iterated function system of Kannan contractions in controlled metric space

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Abstract

The fixed point theory is one of the most essential techniques of applicable mathematics for solving many realistic problems to get a unique solution by using the well known Banach contraction principle. It has paved the ways for numerous extensions, generalization and development of the theory of fixed points in very diverse settings. Our intention in the present paper is to study the Kannan contraction maps defined on a controlled metric space. The generalization of the fixed point theorem for Kannan contraction on controlled metric space is investigated in this paper. We construct an iterated function system called Controlled Kannan Iterated Function System (CK-IFS) with Kannan contraction maps in a controlled metric space and use it to develop a new kind of invariant set, which is called a Controlled Kannan Attractor or Controlled Kannan Fractal (CK-Fractal). Subsequently, the collage theorem for controlled Kannan fractal is also proved. The multivalued fractals are also constructed in the controlled metric space using Kannan and Reich-type contraction maps. The newly developing iterated function system and fractal set in the controlled metric space can provide a novel direction in the fractal theory.

Introduction

The classical geometry was used to study all the human made and natural objects that usually takes into account the concept of Euclidean and topological dimension irrespective of the smoothness or complex nature of the objects. All such objects are believed to be of integer topological dimensions. Mandelbrot [24] observed that the traditional Euclidean geometry is not fully capable of studying various natural objects whose geometric shapes are very complex and irregular such as mountains, clouds, trees, coastlines, fluctuation in stock market etc. He revealed the property of self-similarity as a common feature in all such complex and irregular shapes, signals or systems. He extended the classical geometry to the new notion of fractal geometry, which is believed to be a better option to study, approximate or model such complex objects, shapes or signals etc. The fractal geometry is used to describe complex structures and infinite intricacies of items in the real world. Fractals are often used by researchers today in the various fields of science, such as time evolution of quantum fractals, fractal-time derivative operators, Sierpinski-type fractal structures, fractionally-perturbed systems, quantum mechanics, kinetic energy, topological insulators, and other applications for physical problems.

The theory of Iterated Function Systems (IFS) was Initiated by by Hutchinson [21] and later elaborated by Barnsley to generate fractals. Barnsley has extensively discussed the fixed point theory to generate the deterministic fractals. Moreover, Hutchinson's theory of IFS uses it as a crucial tool, which has been wildly used to construct different types of fractals. Image processing, stochastic dynamic structure, and stochastic growth models are some examples of IFS applications. The famous Banach contraction principle applies to a specific proportion or magnitude of IFS in a complete metric space (CM) [6], [7], [8], [15], [16], [21]. Fractals are often extensively applied in physics and chemistry and other branches of sciences and engineering including study of quantum theory, partially chaotic systems, and fractal-time derivative operators. Researchers nowadays use fractals in a variety of domains and many other aspects of physical issues.

Many researchers use Hutchinson's theory of IFS for further advancement and application of IFS to more general intervals and general contractions, as well as to infinite IFS and multifunctional systems, in order to generate new types of invariant sets with distinct dimensional scales [1], [2], [3], [4], [10], [12], [13], [14], [18], [26], [31], [32], [35], [36], [37], [38], [39], [40]. Hata [20] used the conditional functions to generate IFS and idea of unbounded IFSs was proposed by Bernau [17] and studied by many researchers [19], [25]. Secelean [34] investigated countably recursive functional systems in a condensed metric space and suggested a combination of several contractions into F-contractions to create a new IFS. The idea of a topological IFS attractor, which generalizes the well-known IFS attractor, was developed by many researchers [29], [30]. Also a lot of research work has been done on IFS by considering the Kannan contractions and other type of maps [9], [11], [33].

A controlled metric space (CMS) is a general space extended from the b−metric space. It controls the triangle inequality from the two-variable control function as the control element of the system and many researchers have established the interesting theorems and other results using this metric [27], [28]. The above flow of extensions in this situation prompts us to study the Kannan contraction maps in CMS and construct IFS, to explain HB theory, and to create a new class of fractal sets in the suggested CMS.

The structure of the article is organized as follows. Section 2 discusses the basic concepts of the contraction, Banach contraction principle, Hausdorff metric space, and iterated function system, which are required for this research work. Further, the fixed point theorem for Kannan contraction on complete controlled metric space (CCMS) and other consequent results are proved in Section 3.

The existence of fractals in controlled metric space using the iterated function system of Kannan contractions and other interesting results are established in Section 4. The multivalued structure of fractal is constructed and discussed in Section 5. Finally, the obtained results are concluded in Section 6.

Section snippets

Preliminaries

First, we discuss some concepts of IFS theory which are necessary for our work.

Let (E, d) be a metric space (MS). The function $\Omega : E \rightarrow E$ is known as contraction if Ω fulfills $d\left(\Omega\left(\eta\right), \Omega\left(\zeta\right)\right) \leq sd\left(\eta, \zeta\right), \forall \eta, \zeta \in E, s \in [0, 1)$, where s is the contraction ratio.

Banach contraction principle, a well-known result established by Banach in 1922 [5] which provides a crucial tool in fixed-point theory.

Theorem 1

Let (E, d) be MS and let $\Omega : E \rightarrow E$ be contraction mapping. Then Ω has a unique fixed point. In addition, the sequences of…

…

Kannan contraction mapping on controlled metric space

In this part, we discuss the novel IFS concept by replacing the contraction with a general Kannan contraction.

In 1968, Kannan introduced the contraction called the Kannan contraction [22], [23], viewed as follows.

If there exist a contraction ratio s such that $s \in \left[0, \frac{1}{2}\right)$, and $\forall \eta, \zeta \in E$ and $d\left(\Omega\left(\eta\right), \Omega\left(y\right)\right) \leq s\left[d\left(\eta, \Omega\left(\eta\right)\right) + d\left(\zeta, \Omega\left(\zeta\right)\right)\right]$,then Ω is called the Kannan contraction.

In the flow of extension we introduce the Kannan contraction on the CMS as follows

If (E, d) is a complete controlled metric space (CCMS)…

Controlled Kannan fractal

This section represents the Hutchinson Barnsley theorem for attractor in CCMS and some of the important results.

Theorem 8

Let (E, d) be a CCMS. Let $\Omega : E \to E$ be a continuous Kannan contraction mapping associating contraction ratio $s \in \left(0, \frac{1}{6}\right)$. Then, the mapping $\Omega : \mathscr{K}_0\left(E\right) \to \mathscr{K}_0\left(E\right)$ is characterized as $\Omega\left(Y\right) = \{\Omega\left(\eta\right) : \eta \in X\}$, $\forall X \in \mathscr{K}_0\left(E\right)$, a Kannan contraction on $\left(\mathscr{K}_0\left(E\right), \mathscr{H}_d\right)$ associating contractivity ratio $0 < s = \frac{\beta}{1-\beta} < \frac{1}{2}$….

Proof

Let Ω be a continuous mapping. Thus, Ω is a self map on $\mathscr{K}_0\left(E\right)$ [8]. Let $X, Y \in \mathscr{K}_0\left(E\right)$. Then,
$\mathbb{H}_d\left(\Omega\left(X\right), \Omega\left(Y\right)\right) = d\left(\Omega\left(X\right), \Omega\left(Y\right)\right) \vee d(\Omega$…

…

Controlled multivalued fractals on Kannan contractions

In this section, we discuss multivalued Kannan contractions in controlled metric space for generating multivalued CK-Fractal. Here, $\mathscr{C}\left(E\right)$ denotes the collection of all nonempty closed subsets of E.

Theorem 12

If (E, d) *is a controlled complete metric space,* $\Omega_1, \Omega_2 : E \longrightarrow \mathscr{C}(E)$ *are multi-valued Kannan contractions, which are upper semi-continuous mappings, and the functions* $\psi_1, \psi_2 : \mathbb{R}_+^3 \longrightarrow \mathbb{R}_+$*, then the following conditions are satisfied for* $i \in \{1, 2\}$.

(1) ψ_i *is continuous function which is also increasing.*

(2) $\lim_{n\to\infty} \psi_i^n(\zeta) = 0 \ldots$

…

Conclusion

In this study, a generalization of the fixed point theorem for the Kannan contraction on a controlled metric space was explored. The Kannan contraction over controlled metric spaces was utilized in this study to develop a new type of iterated function system, such as CK-IFS. Essentially, an iterated function system of Kannan contractions has been constructed in a controlled metric space to generate controlled Kannan fractals. The subsequent results were proved interestingly on the controlled…

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