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Ginzburg Landau Model for Nanofluid Convection in the Presence of Time Periodic Plate Modulation

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ABSTRACT

Here we study the thermal modulation effect on nanofluid convection and discuss heat and mass transfer in the layer. The non-uniform time-periodic boundary conditions of the system are considered. A weak non-linear stability analysis has been performed and obtained heat and mass transfer coefficients as a function of the system parameters. The Ginzburg Landau model was employed to derive nanofluid convective amplitude at different stages of flow disturbances and modulation. Slow variations of time scale show that thermal modulation impact on transport phenomenon for the case of out-phase modulation (OPM) and (lower boundary modulation) LBM. Also, the effect of IPM (in-phase modulation) is observed low effect on Nu and Nuc which are similar to the un-modulation case. It is also justified that LBM results are similar to gravity modulation results. It is found that thermal modulation and concentration Rayleigh numbers either stabilize or destabilize the system. Further, the GL model shows better results in the regulation of the transport process.

1. Introduction

Convection in nanofluids is very important to analyze the thermal properties of nanoliquids. Studies related to nanofluids received a lot of interest by many authors due to their variety behavior sudden enhancement in thermal conductivity. Un-expected abnormal behavior of nanofluids got attention to investigate linear and nonlinear flow models. Choi and Eastman [1] was the first person to study nanofluid, referring to fluids containing dimension of the order of tens or hundreds of nanometers. A comprehensive review of heat transport in nanofluids is due to Eastman *et al.,* [2,3]. It is fact that there is no satisfactory reasons that could be found far an abnormal growth rate in the thermal conductivity and viscosity in the presence of nano-particles. The increment of effective thermal conductivity was confirmed by experiments conducted by many researchers, including Masuda *et al.,* [4], although the level of enhancement is still a subject of a debate [5]. The unique properties of nanofluids suggest the possibility of using nanofluids in a variety of engineering systems, from advanced nuclear systems to drug delivery. Other relevant studies of nanofluids and

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their convection instabilities are given by Buongiorno [6], Tzou [7], Nield and Kuznetsov [8], and Kuznetsov and Nield [9].

Natural convection in nanofluids have been investigated by many authors in recent past for different physical configurations [10,11]. It is a common finding of these studies that the presence of nano-particles in the base fluid can advance/delay the onset of convection, and thereby heat transport results, based on concentration gradient of the nano-particles. One interesting finding of these studies is that the instability is purely due to buoyancy force coupled with conservation of nano-particles and is independent of Brownian motion and thermophoresis effects. Thus, one needs to alternate gravity fields along with nanofluids to control instability in the media. In fact, Brownian motion produce their effect only in coupling the temperature and the particle concentration.

No data reported that nanofluids studies for nonlinear modes of convection under modulation. It was Bhadauria and Kiran [12] who introduced modulation effects on nanofluid convection for nonlinear modes. The effect of gravity modulation on nanoconvection was given by Bhadauria and Kiran [12] and Bhadauria *et al.,* [13]. It was found that modulation regulate transport phenomenon with finite amplitude. The study of nonlinear thermal instability in a viscoelastic nanofluid saturated porous medium under gravitational modulation was given by Kiran [14]. The same problem for internal heating was presented in Kiran *et al.,* [15]. The effect of thermal modulation on nanofluid convection was introduced by Kiran and Narasimhulu [16,17]. Here they have found that modulation effect not only controls transport phenomenon but also on chaotic convection. The effect on throughflow on nanofluid convection was given by Kiran *et al.,* [18]. It was found that throughflow shows both in and out flows enhances or diminishes heat and mass transfer in the layer. In the literature till date no data reported Ginzburg Landau (GL) model for nonlinear nanofluid convection under temperature modulation. The GL model is used to find finite amplitude of nonlinear thermal instability. The GL equation is having lot of potential applications in chemical and thermal engineering science.

Regulation of convective flows by external constraints like temperature or gravity modulation has been of great interest due to its immense practical applications in various heat and mass transfer problems. Venezian [19] reported the effect of temperatures modulation which gives rise to an unsteady basic temperature gradient on the onset of Rayleigh-Bénard convection. The effect of gravity modulation, i.e. vertical vibrations, on Rayleigh-Bénard convection was first reported by Gresho and Sani [20]. Other related gravity modulation works recently presented in researches by Kiran [21-23], Kiran and Narasimhulu [24], and Gaikwad *et al.,* [25]. They have discussed the effect of gravity modulation on thermal Rayleigh number and quantified heat and mass transfer in the system.

Finite amplitude convection in nanofluids for nonlinear modes was given by Bhadauria and Agarwal [27], Agarwal *et al.,* [28], and Agarwal [29]. Rotational effects are accounted on nanofluid saturated rotating porous convection. They have used truncated representation of fourier series to convert nonlinear PDE into set of simultaneous ODE's. Their study reported that nanofluids are modeled to enhance thermal and concentration transport. Khalid *et al.,* [30] discussed the control effect on Rayleigh-Benard convection in rotating nanofluids layer with double diffusive coefficients. They have determined the variables which are stabilize and destabilize the system for various boundary conditions. The characteristics of nanofluid in terms of heat and mass transfer over a shrinking cylinder and channel reported by Najib and Bachok [31], Phu *et al.,* [33], and Azman *et al.,* [34]. The abnormal behaviour or nanofluids has been reported in their studies. The other studies which are similar to Umavathi [26], Bhadauria and Agarwal [27], Agarwal *et al.,* [28], and Agarwal [29] are given by Agarwal and Bhadauria [35], Rana and Agarwal [36], and Agarwal and Rana [37] investigated binary nanoconvection. In all the studies most of the results reported their onset convection and transport phenomenon without modulation and different thermal boundary conditions.

Govindarajan *et al.,* [47] investigated modulational instability in a asymmetric dual-core nonlinear directional couplers incorporating the effects of the differences in effective mode areas and group velocity dispersions. Djob *et al.,* [48] applied a non-Lagrangian approach for coupled complex Ginzburg-Landau systems to investigate the chaotic patterns and bifurcation points on the minimum length of the fiber owing to the impact of third-order dispersion. Drissi *et al.,* [49] developed an algorithm named Asymptotic Numerical Method (ANM) with the Spectral Method (SM) to find the resolution of the Ginzburg–Landau envelope equation.

Some more work on thermal modulation in recent times includes; Bhadauria and Kiran [38], Kiran and Bhadauria [39], Kiran *et al.,* [40,41] and Manjula and Kiran [42] are few [43]. These studies introduced external mechanisms to control the convective flow effectively. Following their work, many investigators studied the linear and nonlinear stability of Rayleigh-Bénard convection for different configurations and by using various methods. Umavathi [26] reported that, the effect of temperature modulation on nanofluid convection in porous medium by performing a linear stability analysis. It was reported that the temperature modulation can be used to advance or delay the onset of nanofluid convection. There is no study available which concern to the effect of thermal modulation on nanofluid convection with GL model. Thus, in this paper, the influence of thermal modulation on nanofluid convection has been studied by performing weak nonlinear stability analyses. Consequently, the thermal and concentration Nusselt number were calculated as a function of other physical parameters.

Till today no work reported that discuss GL model on nano-convection with modulation. Thus the present paper is aimed to present the results of GL model on nanoconvection. Consequently, the objective of the current study is to examine the effect of thermal modulation of nanofluid convection under GL model. The solutions are obtained analytically and numerically based on the cartesian coordinate procedure. In this paper we consider the effect of modulation in three different profiles IPM, OPM and LBM. A comparison among three different modulation cases has been observed.

2. Governing Equations

Horizontal layer with nanofluid is confined between two boundaries at z=0 and z=d, heated from below and cooled from above is consider. The boundary layers are considered to be impermeable and perfectly thermally conducting. The nanofluid layer extended infinitely in x and y-directions, and z-axis is taken vertically upward with the origin at the lower boundary. T_h and T_c are the temperatures at the lower and upper walls respectively, the former being greater. A detailed derivation of the conservation equations has been dealt by Buongiorno [6], Tzou [7] and Kuznetsov and Nield [9].

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q}\right) = -\nabla p + \mu \nabla^2 \vec{q} + [\phi \rho_p + (1-\phi)\{\rho_f(1-\beta(T-T_c))\}]\vec{g} \tag{2}$$

$$(\rho c)_f \left[\frac{\partial T}{\partial t} + \vec{q} \cdot \nabla T\right] = k_f \nabla^2 T + (\rho c)_p [D_B \nabla \phi \cdot \nabla T] \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\epsilon} \vec{q} \cdot \nabla \phi = D_B \nabla^2 \phi, \tag{4}$$

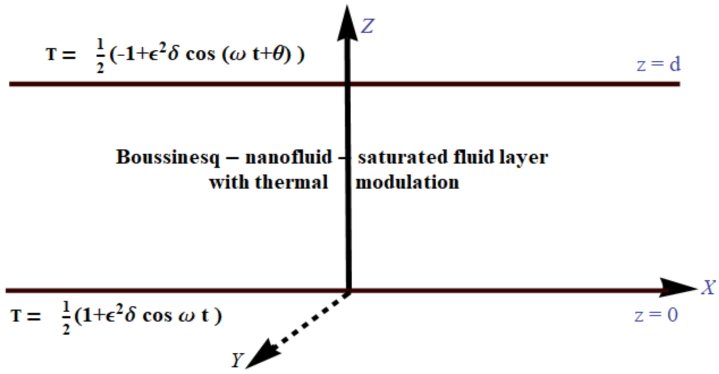

Fig. 1. Physical configuration of gravity modulation

where $\vec{q} = (u, v, w)$ is the fluid velocity. In these equations, ρ is the fluid density, $(\rho c)_f$, $(\rho c)_p$, the effective heat capacities of the fluid and particle phases respectively, k_f the effective thermal conductivity of fluid phase, D_B denote the Brownian diffusion coefficient. We assume temperature and volumetric fraction (of nano-particles) to be constant at stress-free plates, we impose the initial condition on T and ϕ as:

$$\vec{q} = 0, \quad T = T_h, \ \phi = \phi_0 \quad \text{at} \quad z = 0, \tag{5}$$

$$\vec{q} = 0, \quad T = T_c, \ \phi = \phi_1 \quad \text{at} \quad z = d. \tag{6}$$

The following externally applied boundary conditions are considered in this paper [19]:

$$\begin{aligned} T &= \tfrac{1}{2}[1 + \epsilon^2\delta\cos(\omega t)] \quad 1.1 inat \ z = 0 \\ &= \tfrac{1}{2}[-1 + \epsilon^2\delta\cos(\omega t + \theta)] \quad 1.0 inat \ z = d \end{aligned} \tag{7}$$

where δ and Ω are the amplitude frequency of modulation, θ is the phase difference. To non-dimensionalize the physical variables we take following transformations:

$$(x^*, y^*, z^*) = \frac{(x,y,z)}{d}, \ t^* = \frac{t\alpha_f}{d^2}, T^* = \frac{T - T_c}{T_h - T_c},$$

$$(u^*, v^*, w^*) = \frac{(u,v,w)d}{\alpha_f}, \ p^* = \frac{pd^2}{\mu\alpha_f}, \phi^* = \frac{\phi - \phi_0}{\phi_1 - \phi_0},$$

where $\alpha_f = \frac{k_f}{(\rho c)_f}$. Using the above transformations into Eq. (1) to Eq. (6) we obtain the following dimensionless governing system (after dropping the asterisk):

$$\nabla \cdot \vec{q} = 0, \tag{8}$$

$$\frac{1}{Pr}\left(\frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q}\right) - \nabla^2 \vec{q} = -\nabla p + (Ra_T T + Rn \ \phi - Rm)\vec{k}, \tag{9}$$

$$\frac{\partial T}{\partial t} + \vec{q} \cdot \nabla T = \nabla^2 T + \frac{N_B}{Le} \nabla\phi \cdot \nabla T, \tag{10}$$

$$\frac{\partial \phi}{\partial t} + \vec{q} \cdot \nabla \phi = \frac{1}{Le} \nabla^2 \phi, \tag{11}$$

$$\vec{q} = 0, \quad \phi = 0 \;\; at \;\; z = 0, \tag{12}$$

$$\vec{q} = 0, \quad \phi = 1 \;\; at \;\; z = 1. \tag{13}$$

In the above system, the non-dimensional variables have their usual meanings (presented in nomenclature).

In the state of motionless fluid the disturbances are at rest and the quantities vary in z-direction:

$$\vec{q} = 0, \;\; p = p_b(z,t) \;\; T = T_b(z) \;\; \phi = \phi_b(z). \tag{14}$$

Substituting Eq. (14) in Eq. (10) and Eq. (11), we get

$$\frac{d^2T_b}{dz^2} + \frac{N_B}{Le}\frac{d\phi_b}{dz}\frac{dT_b}{dz} = 0, \tag{15}$$

Using an order of magnitude analysis, Kuznetsov and Nield [9] showed that the second and third terms in Eq. (15) are small and hence we have:

$$\frac{d^2T_b}{dz^2} = 0, \quad \frac{d^2\phi_b}{dz^2} = 0 \tag{16}$$

The following boundary conditions are taken to solve the Eq. (16) from the Eq. (12) and Eq. (13):

$$T_b = 1, \;\; \phi_b = 0 \;\; at \;\; z = 0, \tag{17}$$

$$T_b = 0, \;\; \phi_b = 1 \;\; at \;\; z = 1. \tag{18}$$

The following is obtained while solving the Eq. (16), subject to the given condition Eq. (17) and Eq. (18):

$$\phi_b = z \tag{19}$$

The basic state equations are superimposed by perturbations as given bellow:

$$\vec{q} = \vec{q}\,', \;\; p = p_b + p', \;\; T = T_b + T', \;\; \phi = \phi_b + \phi'. \tag{20}$$

The following is obtained while substituting the Eq. (20) in Eq. (8) to Eq. (11) and using the expressions Eq. (19):

$$-\nabla^4\psi + \frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi) = \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)} - \left(Ra_T\frac{\partial T}{\partial x} - Rn\frac{\partial\phi}{\partial x}\right) \tag{21}$$

$$\frac{\partial T_b}{\partial z}\frac{\partial\psi}{\partial x} - \nabla^2 T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi,T)}{\partial(x,z)} \tag{22}$$

$$-\frac{\partial\psi}{\partial x} - 0 = \frac{1}{Le}\nabla^2\phi - \frac{\partial\phi}{\partial t} + \frac{\partial(\psi,\phi)}{\partial(x,z)} \tag{23}$$

The above system is solved using the thermal boundary conditions on the plates z=0 and z=d. From the Eq. (22) the term $\frac{\partial T_b}{\partial z}$ influences the stability problem through subject to the Eq. (7):

$$\frac{\partial T_b}{\partial z} = -1 + \epsilon^2 \delta [f_2(z,t)], \tag{24}$$

where

$$f_2 = \mathrm{Re}[f(z) e^{-i\omega t}], \tag{25}$$

$$f(z) = [A(\lambda) e^{\lambda z} + A(-\lambda) e^{-\lambda z}], A(\lambda) = \frac{\lambda}{2} \frac{(e^{-i\theta} - e^{-\lambda})}{(e^{\lambda} - e^{-\lambda})} \text{ and } \lambda = (1-i)\sqrt{\frac{\omega}{2}}.$$

A small variation of time is considered and it is re-scaled as $\tau = \epsilon^2 t$. With this assumption stationary convection of the system will be investigated. The non-linear system of equations Eq. (21) to Eq. (23) are expressed in the matrix form:

$$\begin{bmatrix} -\nabla^4 & Ra_T \frac{\partial}{\partial x} & -Rn \frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z} \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ -\frac{\partial}{\partial x} & 0 & -\frac{1}{Le} \nabla^2 \end{bmatrix} \begin{bmatrix} \psi \\ T \\ \phi \end{bmatrix} = \begin{bmatrix} -\frac{1}{Pr} \frac{\partial}{\partial \tau} (\nabla^2 \psi) + \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x,z)} \\ -\frac{\partial T}{\partial \tau} + \frac{\partial(\psi, T)}{\partial(x,z)} \\ -\frac{\partial \phi}{\partial \tau} + \frac{\partial(\psi, \phi)}{\partial(x,z)} \end{bmatrix} \tag{26}$$

The equations given in Eq. (26) are solved subject to stress-free, isothermal, iso-nanoconcentration boundary conditions:

$$\psi = \nabla^2 \psi = T = \phi = 0 \text{ at } z = 0,1 \tag{27}$$

3. Heat Mass Transport

The following system of asymptotic expansions substituted in Eq. (26):

$$\begin{aligned} Ra_T &= R_{0c} + \epsilon^2 R_2 + \epsilon^4 R_4 + \ldots, \\ \psi &= \epsilon \psi_1 + \epsilon^2 \psi_2 + \epsilon^3 \psi_3 + \ldots, \\ T &= \epsilon T_1 + \epsilon^2 T_2 + \epsilon^3 T_3 + \ldots, \\ \phi &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \ldots, \end{aligned} \tag{28}$$

where R_{0c} is the critical value of the Rayleigh number at which the onset of convection takes place in the absence of temperature modulation. Now we solve the system for different orders of ϵ.

At the lowest order: The lowest order system is given by:

$$\begin{bmatrix} -\nabla^4 & R_{0c} \frac{\partial}{\partial x} & -Rn \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\nabla^2 & 0 \\ -\frac{\partial}{\partial x} & 0 & -\frac{1}{Le} \nabla^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \\ \phi_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{29}$$

The solutions of the lowest order system subject to the boundary conditions Eq. (27) is:

$$\psi_1 = \mathbb{B}(\tau)\sin(k_c x)\sin(\pi z), \tag{30}$$

$$T_1 = -\frac{k_c}{\delta^2}\mathbb{B}(\tau)\cos(k_c x)\sin(\pi z), \tag{31}$$

$$\phi_1 = \frac{k_c}{\delta^2}(Le)\mathbb{B}(\tau)\cos(k_c x)\sin(\pi z), \tag{32}$$

where $\delta^2 = k_c^2 + \pi^2$. The critical value of the Rayleigh number and the corresponding wave number for the onset of stationary convection is calculated numerically and the expression are given by

$$R_{0c} = \frac{\delta^6}{k_c^2} - Rn(Le),$$

$$k_c = \frac{\pi}{\sqrt{2}},$$

which are the results given by Venezian [19] and Gresho and Sani [20] for normal fluids.

At the second order: In this order the following system of equations is obtained:

$$\begin{bmatrix} -\nabla^4 & R_{0c}\frac{\partial}{\partial x} & -Rn\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\nabla^2 & 0 \\ -\frac{\partial}{\partial x} & -0 & -\frac{1}{Le}\nabla^2 \end{bmatrix}\begin{bmatrix}\psi_2 \\ T_2 \\ \phi_2\end{bmatrix} = \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23}\end{bmatrix} \tag{33}$$

$$R_{21} = 0, \tag{34}$$

$$R_{22} = \frac{\partial \psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial T_1}{\partial x}, \tag{35}$$

$$R_{22} = \frac{\partial \psi_1}{\partial x}\frac{\partial \phi_1}{\partial z} - \frac{\partial \psi_1}{\partial z}\frac{\partial \phi_1}{\partial x}. \tag{36}$$

The second order solutions subjected to the boundary conditions Eq. (27) is obtained as follows:

$$\psi_2 = 0 \tag{37}$$

$$T_2 = -\frac{k_c^2}{8\pi\delta^2}\mathbb{B}^2(\tau)\sin(2\pi z), \tag{38}$$

$$\phi_2 = \frac{k_c^2}{8\pi\delta^2}(Le)\mathbb{B}^2(\tau)\sin(2\pi z). \tag{39}$$

The horizontally averaged Nusselt number, $Nu_f(\tau)$, for the stationary mode of convection is given by:

$$Nu(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}\left(\frac{\partial T_2}{\partial z}\right)dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}}\left(\frac{\partial T_b}{\partial z}\right)dx\right]_{z=0}} \tag{40}$$

$$Nu(\tau) = 1 + \frac{k_c^2}{4\delta^2}\mathbb{B}^2(\tau). \tag{41}$$

The nanoparticle concentration Nusselt number, $Nu_c(\tau)$ is defined similar to the thermal Nusselt number. Following the procedure adopted for arriving at Nu, one can obtain the expression for Nu_c in the form:

$$Nu_c(\tau) = 1 + \frac{k_c^2}{4\delta^2}(Le))\mathbb{B}^2(\tau). \tag{42}$$

At the third order: In this order the following system is obtained:

$$\begin{bmatrix} -\nabla^4 & R_{0c}\frac{\partial}{\partial x} & -Rn\frac{\partial}{\partial x} \\ -\frac{\partial T_b}{\partial z}\frac{\partial}{\partial x} & -\nabla^2 & 0 \\ -\frac{\partial}{\partial x} & 0 & -\frac{1}{Le}\nabla^2 \end{bmatrix}\begin{bmatrix}\psi_3 \\ T_3 \\ \phi_3\end{bmatrix} = \begin{bmatrix}R_{31} \\ R_{32} \\ R_{33}\end{bmatrix} \tag{43}$$

where

$$R_{31} = -\frac{1}{Pr}\frac{\partial}{\partial\tau}(\nabla^2\psi_1) - R_2\frac{\partial T_1}{\partial x} - R_{0c}\frac{\partial T_2}{\partial x} + Rn\frac{\partial\phi_2}{\partial x}, \tag{44}$$

$$R_{32} = -\frac{\partial T_1}{\partial\tau} + \frac{\partial\psi_1}{\partial x}\frac{\partial T_2}{\partial z} + \delta_1 f_2\frac{\partial\psi_1}{\partial x}, \tag{45}$$

$$R_{33} = -\frac{\partial\phi_1}{\partial\tau} + \frac{\partial\psi_1}{\partial x}\frac{\partial\phi_2}{\partial z} \tag{46}$$

Substituting ψ_1, T_1 and T_2 into Eq. (44) to Eq. (46), we can obtain expressions for R_{31}, R_{32} and R_{33} easily. Now by applying the solvability condition for the existence of third order solution, we get the Ginzburg-Landau equation for stationary convection with time-periodic coefficients in the form:

$$A_1\mathbb{B}'(\tau) = A_2\mathbb{B}(\tau) - A_3\mathbb{B}(\tau)^3 \tag{47}$$

where

$A_1 = \frac{\delta^2}{Pr} + \frac{k_c^2}{\delta^4}(R_{0c}) + \frac{Rnk_c^2Le^2}{\delta^4}, A_2 = \left[\frac{R_2k_c^2}{\delta^2} + \left(R_{0c}k_c^2\delta^2 + \frac{Rnk_c^2}{\delta^2}I_1\right)\right], A_3 = \frac{k_c^4}{8\delta^4}[R_{0c} - RnLe^3], I_1 = \int_0^{\frac{2\pi}{k_c}}\sin^2(\pi z)dz.$

The non autonomous Ginzburg-Landau equation Eq. (47) is a Bernoulli equation with time variate coefficients and obtaining its analytical solution is difficult [49-52]. This equation is solved numerically with NDSolve of Mathematica, subjected to initial value $\mathbb{A}(0) = a_0$, where a_0 for initial amplitude. Here we choose $R_2 = R_{0c}$, for weakly nonlinear convection.

4. Solutions without Modulation

In the case of unmodulated system, the above Ginzburg–Landau equation can be written as

$$A_1\mathbb{B}_u{}'(\tau) - A_2\mathbb{B}_u(\tau) + A_3\mathbb{B}_u(\tau)^3 = 0, \tag{48}$$

where $\mathbb{B}_u(\tau)$ is an amplitude of convection for unmodulated case and A_1, A_3 have the same expression as given in the Eq. (47) and $A_2 = \frac{R_{0c}k_c^2}{\delta^2}$. The solution of Eq. (48) is given by

$$\mathbb{B}_u(\tau) = \frac{1}{\sqrt{\left(\frac{A_3}{2A_2}+C_1 Exp\left[\frac{-2A_2}{A_1}\right]\right)}}, \tag{49}$$

where C_1 is a parameter, it can be calculated for given suitable initial condition. The thermal and nanoparticle concentration Nusselt number in this case is obtained from Eq. (41) to Eq. (42) by using amplitude of convection as $\mathbb{B}_u(\tau)$.

5. Results and Discussion

The effect of thermal modulation on the Rayleigh Benard nano-convection is discussed by performing a weakly nonlinear stability analyses. The modulation of Rayleigh–Bénard system has been assumed to be of order $O(\epsilon^2)$, which shows that, we consider only small amplitude of thermal modulation. This assumption leads to obtain an amplitude equation in a simple manner, and much easier than the Lorenz model. It is well known that, making a nonlinear theory to analyze heat mass transport, which is not possible by linear theory. Moreover external regulations of convection is important in the study of thermal instability, therefore, in this paper we have considered thermal modulation for either enhancing or inhibiting convective heat mass transport as is required by application.

The effect of thermal modulation on heat mass transport has been depicted graphically in Figure 2 to Figure 7. In this paper we consider three different types of modulations. We consider three different thermal boundary conditions to see temperature modulation effect on Nu and Nu_c i.e., heat and concentration transport.

The following boundary conditions are considered:

(i) IPM -in phase modulation $\theta = -I\infty$
(ii) OPM -out phase modulation $\theta = \pi$
(iii) LBM -lower boundary modulation $\theta = 0$

The following parameters Pr, Rn, Le, δ and Ω are occurred in our study which influence the convective heat mass transport. The fluid layer is not considered to be highly viscous, therefore only moderate values of Pr are taken for calculations. Because of small amplitude modulation, the values of δ are considered to be small. Further, thermal modulation has been assumed to be of low frequency, as at low range of frequencies, the effect of modulation frequency on onset of convection as well as on heat transport is maximum. A weak nonlinear stability analysis of nanofluids is performed based on Ginzburg-Landau model at third order. The coefficient of heat transports, i.e., thermal Nusselt numbers Nu, and the coefficient of nanoparticle concentration transport Nu_c, i.e., concentration Nusselt number are calculated as functions of time and other system parameters. Our results are depicted in the Figure 2 to Figure 7 for $Nu(\tau)$, $Nu_c(\tau)$.

Stability curves of nano-Rayleigh number are depicted in the Figure 2. The effect of Le and Rn has been identified on onset convection. For linear theory of onset convection it is found that effect of Le and Rn delays the onset convection. The reason for this is due to sudden raise in $R0_c$ there is a delay of onset linear convection. In general when both parameters varies $R0_c$ also varies and become very high which required more energy to move fluid fast. The effect of the diffusivity ratio Le and concentration Rayleigh number is to encourage onset in the system given in Figure 2(a) and Figure 2(b). The corresponding results may observe from the results of Bhadauria and Kiran [44], and Kiran [45,50].

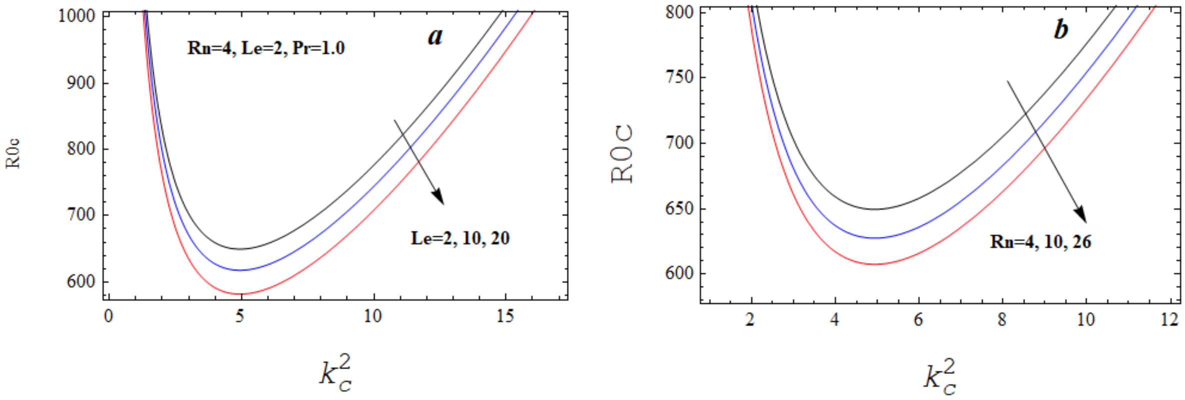

Fig. 2. Stability curves ($R0_c$ versus k_c^2)

The Prandtl number Pr is to enhance heat and concentration transport for low values of time, and further increment in time similar effects can be seen given in Figure 3(a) and Figure 4(a). The influence of concentration Rayleigh number Rn on both thermal and concentration Nusselt numbers is similar, which is to enhance the heat and concentration transport given in Figure 3(b) and Figure 4(b). Thus Rn has dual role on transport media, which can be used to regulate heat mass transfer. Most of the results related to Rn is followed by Agarwal *et al.,* [29-37]. The reader may note that negative values of Rn influence revers nature of heat mass transport in the layer.

To avoid repetition of the graph its graph excluded. Figure 3(c) shows that, an increment in δ is to enhance heat transport. Figure 3(d) reveals the effect of modulation frequency on the thermal Nusselt number $Nu(\tau)$. It can be found that increase in ω reduces the heat transport which is the result reported by Kiran and Narasimhulu [17], Umavathi [26], and Gresho and Sani [20]. The similar results obtained for concentration Rayleigh number so we have presented in Figure 4(c) and Figure 4(d). Moreover, as reported earlier by Bhadauria and Agarwal [27] that Le do not have significant effect on the thermal Nusselt number. Thus we avoid graphical representation of the same. On the contrary in the case of concentration Nusselt number both Le have increasing effect on $Nu_c(\tau)$ as depicted in the Figure 5(a) and Figure 5(b).

Eq. (50) gives an amplitude of convection analytically for unmodulated case, using this in Figure 5(c) and Figure 5(d) we made comparison between modulated and un-modulated system, where unmodulated system shows that, there is an sudden increment in $Nu(\tau)$ and $Nu_c(\tau)$ for small values of time τ and becomes study for large values of time τ. But, incase of modulated system it shows oscillatory behaviour for both $Nu(\tau)$ and $Nu_c(\tau)$. Here the reader may note that only results of $Nu_c(\tau)$ have presented.

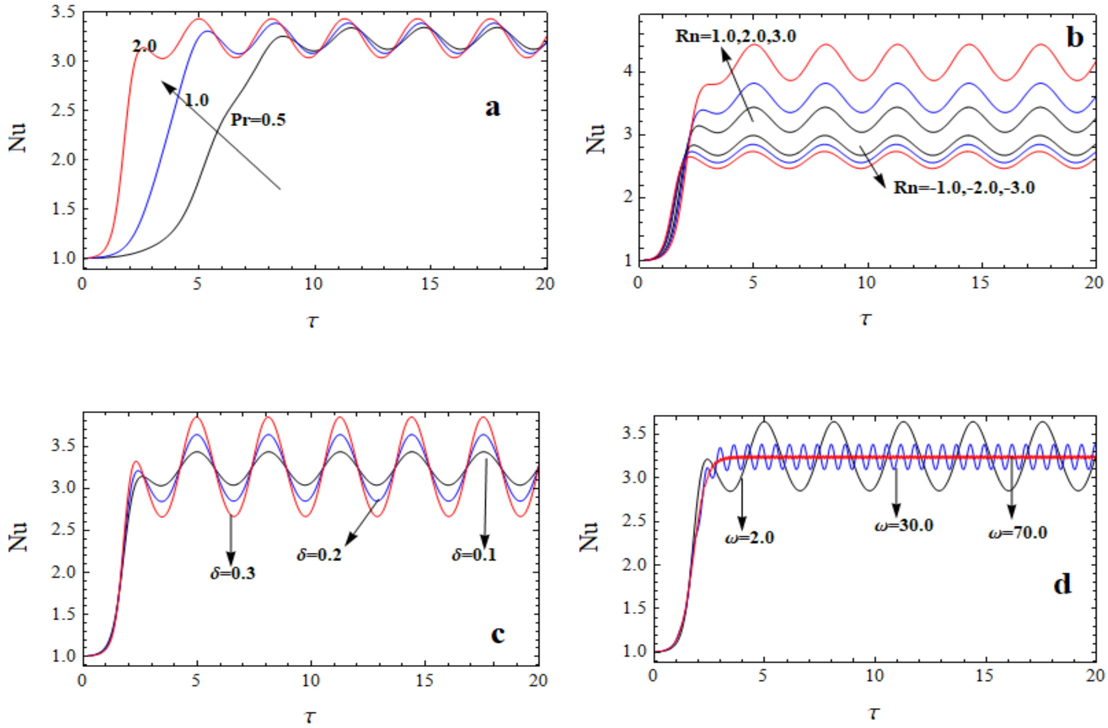

Fig. 3. Heat transfer results based on the effect of (a) Pr (b) Rn (c) δ (d) ω

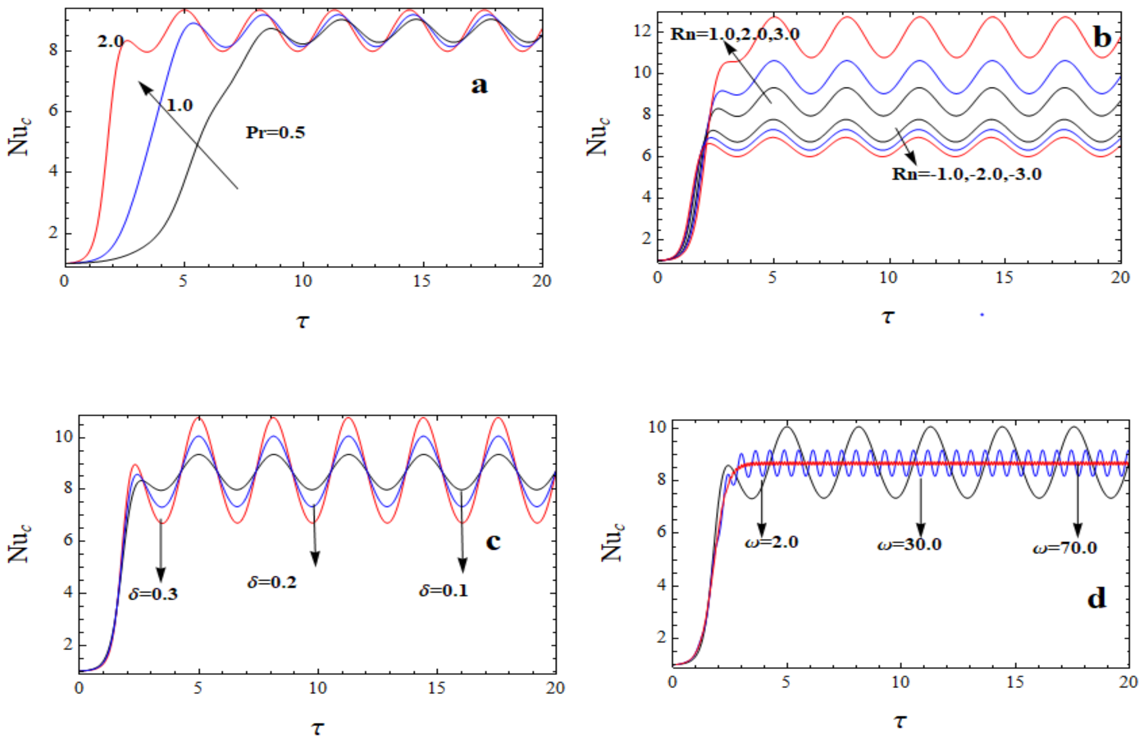

Fig. 4. Nanofluid mass transfer results based on the effect of (a) Pr (b) Rn (c) δ (d) ω

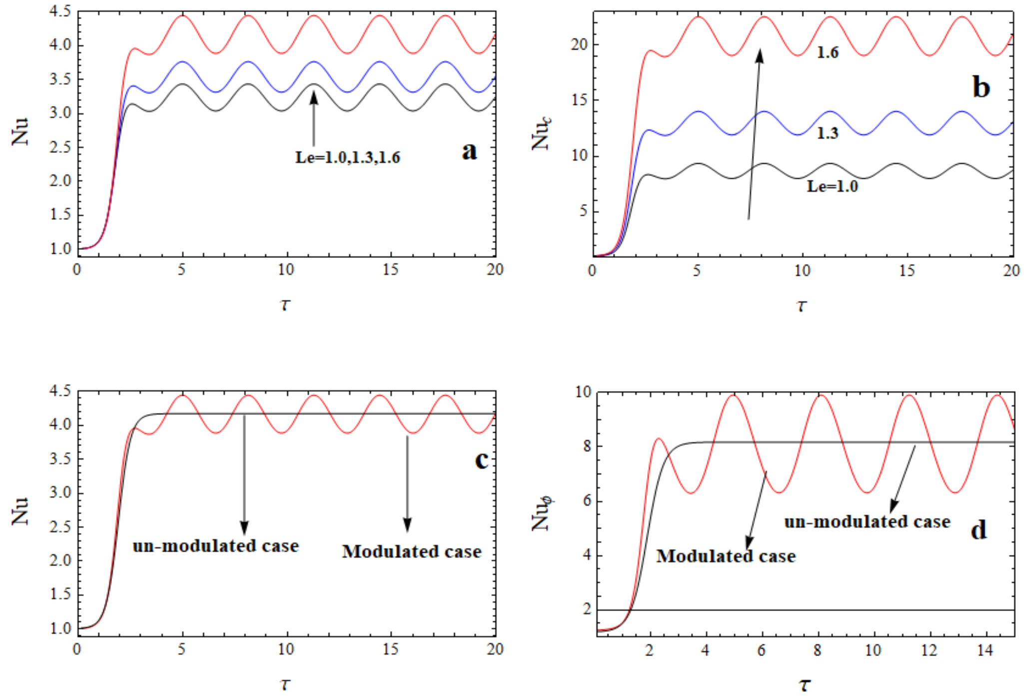

Fig. 5. Heat transfer results based on the effect of (a,b) Le and comparison of modulated and un-modulated case (c,d)

In Figure 6(a) to Figure 6(c) and Figure 7(a) to Figure 7(c) we have presented results corresponding to IPM case. For in phase modulation case the results obtained here of similar nature of those who obtained earlier by Bhadauria and Kiran [38,52], Manjula and Kiran [42] and Kiran and Bhadauria [46], Kiran [50], and Kiran *et al.,* [51]. The comparison of three types of modulation i.e IPM, OPM and LBM cases presented in Figure 6(d) and Figure 7(d). It is found that OPM case shows modulation effect than other two cases. Thus, OPM case is the one which enhances heat mass transfer more in the system.

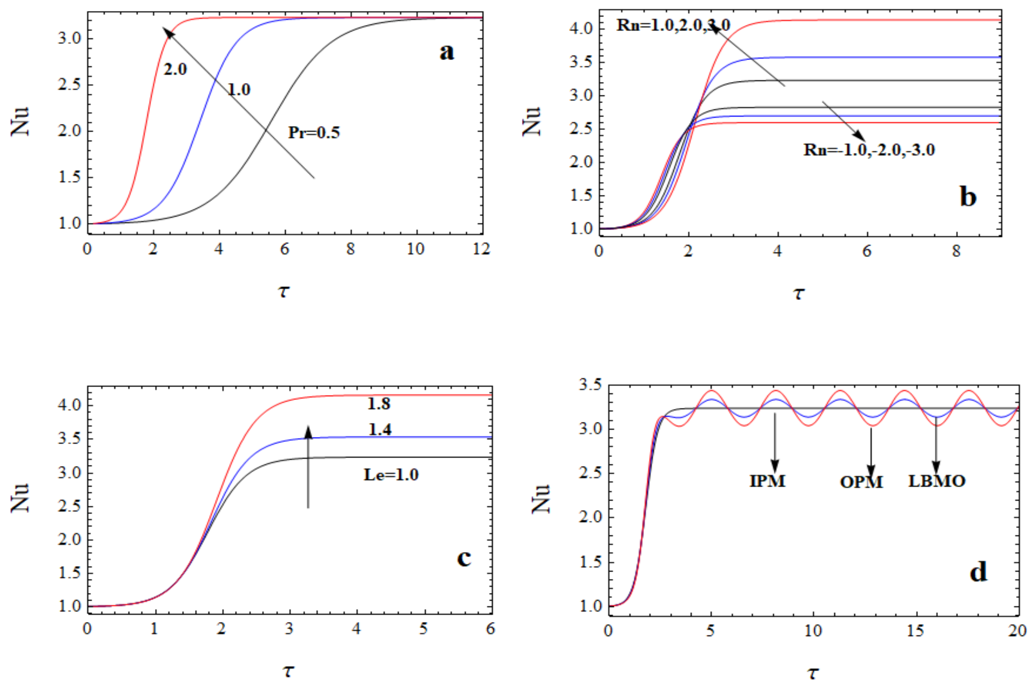

Fig. 6. In phase modulation, results on Nu with respect to different parameters (a) Pr (b) Rn (c) Le (d) IPM, OPM, LBMO

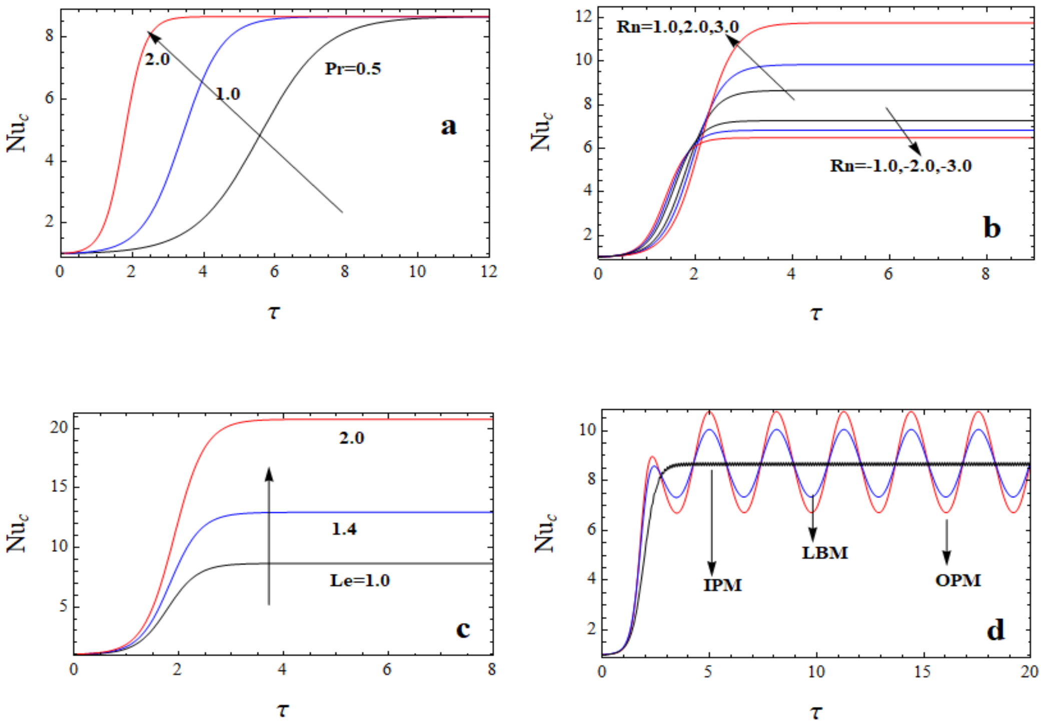

Fig. 7. In phase modulation, Results on Nu_c with respect to different parameters (a) Pr (b) Rn (c) Le (d) IPM, OPM, LBMO

6. Conclusions

The weakly nonlinear stability of a layer of nanofluid is investigated with thermal modulation. The layer is heated from below and cooled from above. We further incorporate the effect of Brownian motion along with the top-heavy suspension of nano particles. The results have been obtained in terms of the concentration and thermal Nusselt numbers with the help of the GL equation. Thus, the effect of various parameters has been obtained and depicted graphically. We have the following observations:

Thermal modulation can be used to control or regulate heat mass transport effectively in the system. Our analysis says that positive values of nanoparticle concentration Rayleigh number Rn, Prandtl number Pr and Lewis number Le enhance the effect of modulation, whereas negative values of Rn show less effect of modulation. The effect of modulation in terms of δ and ω is clear and shows the impact on heat mass transfer. Different types of modulations OPM, LBM, and IPM are reported on heat mass transfer. It shows that OPM cases transfer better results than IPM and LBM cases. Among the three the least one is IPM, which acts like an un-modulated case. The corresponding results may summarise as $[Nu/Nuc]OPM$>$[Nu/Nuc]LBM$>$[Nu/Nuc]IPM$. Finally, it is concluded that by suitable adjustment of three parameter values θ, δ, and ω one may have control over heat mass transfer.

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