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Investigation on Prime Quasi-Ideals in Tg-semirings

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Abstract: We explored prime and semi-prime quasi-ideals in TGSRs and give its portrayal in this article. Likewise, we demonstrate that a quasi-ideal P of a TGSR R will be R-prime ⇔KΓMΓL ⊆ P ⟹ K ⊆ P or M ⊆ P or L ⊆P for any right ideal K, medial ideal M and left ideal L of R.

Key words: Ternary gamma semi-ring, Quasi-ideals and Prime quasi-ideals.

INTRODUCTION AND PRELIMINARIES

A bi-ideal and semi ideal in ternary semi rings was presented by S. Kar [5] and got their properties. G. Srinivasa Rao et.al [10-17] explored and studies such a great amount on ternary semi rings and requested ternary semi rings. We explored by the designs of prime and semi prime semi standards in ternary gamma semi rings, in this composition. For starters allude the references [10-17]. A non-void subset I of a ternary Γ-semi-ring R is supposed to be *left* (*lateral, right*) *ternary Γ-ideal* of R, if (1) $a, b \in I \Rightarrow a + b \in I$ (2) $a, b \in R$, $i \in I$, $\alpha, \beta \in \Gamma \Rightarrow a\alpha b\beta i \in I$ ($a\alpha i\beta b \in I$, $i\alpha a\beta b \in I$).An optimal I is supposed to be ternary Γ-ideal, in case it is left, medial and right Γ-ideal of R. Leave R alone a TGS and $\phi \neq B \subseteq R$. The set B said to be a bi-ideal (BI) of R in case S is a TGSSR of R and BΓRΓBΓRΓB ⊆ B. Every element in a TGSR R is an idempotent [12], then R is called *idempotent TGSR*.

Suppose a ∈R, where R is a TGSR. Then the principal
(i) *left ideal generated by a* is given by $\langle a\rangle_l = \{\sum x_i \alpha y_i \beta a + ma / x_i, y_i \in R, m=1, 3, 5, \ldots,$ and $\alpha, \beta \in \Gamma\}$,
(ii*) right ideal generated by a* is given by $\langle a\rangle_r = \{\sum a\alpha x_i \beta y_i + ma / x_i, y_i \in R, m=1, 3, 5, \ldots,$ and $\alpha, \beta \in \Gamma\}$,
(iii) *lateral ideal generated by a* is given by $\langle a\rangle_m = \{\sum x_i \alpha a \beta y_i + p_i \gamma q_i \delta a \mu r_i \nu s_i + ma / p_i,$

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q_i, x_i, y_i, r_i, $s_i \in R$, m=1, 3, 5, ….,$\alpha, \beta, \gamma, \delta, \mu, \nu \in \Gamma$}, where $\sum$ represents the finite sum and is the set of all non-negative odd integers.

If for x, *y*, *z*∈*R*, α, β∈ Γ, $x\alpha y\beta z = 0 \Rightarrow x = 0$ or $y = 0$ or $z = 0$, then a TGSR R is known as *zero divisor free*. A TGSR R is said have

(i) *right cancellative* w. r. t. ternary multiplication (RCM) if $x\alpha a\beta b = y\alpha a\beta b \Rightarrow x = y$

(ii) *laterally cancellative* under ternary multiplication (LLCM) if $a\alpha x\beta b = a\alpha y\beta b \Rightarrow x = y$.

(iii) *left cancellative with respect to multiplication* (LCM) if $a\alpha b\beta x = a\alpha b\beta y \Rightarrow x = y$.

A TGSR R is called cancellative w. r. t. multiplication (CM) if it is LCM, RCM, and LLCM. A cancellative w. r. t. multiplication (CM) TGSR R is Zero divisor free. A TGSR R with at least 2 elements is known as (TDGSR) ternary division gamma semi ring if $0 \neq a$ of R, $\exists\, 0 \neq b \in$ R, α, β ∈ Γ $\ni a\alpha b\beta x = b\alpha a\beta x = x\alpha a\beta b = x\alpha b\beta a = a\alpha x\beta b = b\alpha x\beta a = x$ for all $x \in R$.

PRIME QUASI-IDEALS IN TERNARY GAMMA SEMIRINGS (TGSRS)

Def. 2.1: A quasi-ideal (QI)$\{0\} \neq B \subseteq R$ of a TGSRR is *prime* if $B_1\Gamma B_2\Gamma B_3 \subseteq B \Rightarrow B_1 \subseteq B$ or $B_2 \subseteq B$ or $B_3 \subseteq B$ for any quasi-ideals B_1,B_2 and B_3 of R. AQIB ≠ R of R is semi-prime if $B_1\Gamma B_1\Gamma B_1 \subseteq B \Rightarrow B_1 \subseteq B$ for any QIB_1 of R.

Note2.2: *A prime quasi-ideal (PQI) of a TGSRR is a semi-prime quasi-ideal (SPQI) of R. But every SPQI need not be PQI of R.* This can be observed in the following example.

Ex. 2.3: Let R = Γ = $\mathcal{M}_2$(Z\N), a TGSR of square matrices with 2nd order over Z\N. Let X = $\left\{\begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} : x \in Z\backslash N\right\}$. Then X, a SPQI of R. But X is not a PQI of R, since P = $\left\{\begin{bmatrix} 0 & y \\ 0 & 0 \end{bmatrix} : y \in Z\backslash N\right\}$, Q = $\left\{\begin{bmatrix} 0 & 0 \\ 0 & z \end{bmatrix} : z \in Z\backslash N\right\}$ andS= $\left\{\begin{bmatrix} 0 & 0 \\ u & 0 \end{bmatrix} : u \in Z\backslash N\right\}$are QIs of R such that PΓQΓS ⊆ X but P ⊈ X,Q ⊈ X and S ⊈ X.

Def. 2.4: A QIP≠R of a TGSRR is called *weakly prime quasi-ideal* (WPQI) if P ⊆ A,P ⊆ B,P ⊆ C and AΓBΓC ⊆ P⟹A = P or B = P or C = P for any QIs A,B and C of R.

Note **2.5:** A PQI of a TGSRR is a WPQI of R. Converse need not be true. This can be observed in the following example:

Ex. 2.6: Let R =Γ = $\mathcal{M}_2$(Z\N), a TGSR of 2nd order square matrices over Z\N. Let X = $\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in 30(Z\backslash N)\right\}$. Then X is WPQI of R. But X is not PQI of R, since P = $\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in 2(Z\backslash N)\right\}$, Q=$\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in 3(Z\backslash N)\right\}$, S= $\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in 5(Z\backslash N)\right\}$ are QIs of R such that PΓQΓS ⊆ X.But P ⊈ X,Q ⊈ X and S ⊈ X.

Th. 2.7: If the QIs of TGSR R forms a chain with respect to set inclusion, then each WPQI is a PQI.

Pf.: Let B be a WPQI of R. Let P, Q and S be QIs of R ∋ PΓQΓS ⊆ B. Suppose P ⊈ B, Q ⊈ B and S ⊈ B. By the given data, B ⊆ P, B ⊆ Q and B ⊆ S. Since B is weakly prime, we have P = B or Q = B or S = B, a contradiction. Therefore, P ⊆ B or Q ⊆ B or S ⊆ B. Hence B is a PQI of R.

Prop. 2.8: Suppose R, a TGSR and *a* in R. Then the principal quasi-ideal generated by *a* is given by $\langle a\rangle_q$ = {[*a*ΓRΓR ∩ (RΓ*a*ΓR+RΓRΓ*a*ΓRΓR) ∩ RΓRΓ*a*]+*ma*: *m*∈{1, 3, 5, ….}}.

Pf.: For the proof of Prop.2.8, see the reference [13, Theorems 3.12, 4.12, 5.12]

Prop. 2.9: If B is a prime, then B is a either left or medial or right ideal of R, where R is a TGSR.

Pf.: Given B is a PQI of R. We have (BΓRΓR)Γ(RΓBΓR + RΓRΓBΓRΓR)Γ(RΓRΓB) ⊆ BΓRΓR∩(RΓBΓR + RΓRΓBΓRΓR) ∩RΓRΓB⊆B. Since, B is prime, we have BΓRΓR ⊆ B or RΓBΓR +RΓRΓBΓRΓR ⊆ B or RΓRΓB ⊆ B.Thus, B is a right or medial or left ideal of R.

Prop.2.10: Suppose R, a TGSR and B, a QI of R. Then B is prime ⟺ [(xΓRΓR \ (RΓxΓR + RΓRΓxΓRΓR) \ RΓRΓx)+ mx]Γ[(yΓRΓR \ (RΓyΓR + RΓRΓyΓRΓR) \ RΓRΓy)+my] Γ[(zΓRΓR \ (RΓzΓR + RΓRΓzΓRΓR) \ RΓRΓz)+mz] ⊆ B⟹ x ∈B or y ∈B or z ∈B.

Pf.: Suppose B is a PQI of R and let [(xΓRΓR \ (RΓxΓR + RΓRΓxΓRΓR) \ RΓRΓx)+mx] Γ[(yΓRΓR \ (RΓyΓR + RΓRΓyΓRΓR) \ RΓRΓy)+my] Γ [(zΓRΓR \ (RΓzΓR + RΓRΓzΓRΓR) \ RΓRΓz)+mz]⊆ B for some x, y, z ∈ R. Clearly, [(xΓRΓR \ (RΓxΓR + RΓRΓxΓRΓR) RΓRΓx)+mx], [(yΓRΓR \ (RΓyΓR + RΓRΓyΓRΓR) \ RΓRΓy)+my] and [(zΓRΓR \ (RΓzΓR + RΓRΓzΓRΓR) \ RΓRΓz)+mz]⊆ B are QIs of R. Since B is prime, we have [(xΓRΓR \ (RΓxΓR + RΓRΓxΓRΓR) \ RΓRΓx)+mx] ⊆B or [(yΓRΓR \ (RΓyΓR + RΓRΓyΓRΓR) \ RΓRΓy)+my] ⊆B or[(zΓRΓR \ (RΓzΓR + RΓRΓzΓRΓR) \ RΓRΓz)+mz] ⊆ B. If {xΓRΓR \ (RΓxΓR + RΓRΓxΓRΓR) \ RΓRΓx}+mx ⊆ B, ⟹$\langle x \rangle_q$⊆B⟹ x ∈ B. Using the same procedure, it is easy to prove y ∈ B or z ∈ B.Obviously converse part is true.

Th.2.11: Suppose R, a TGSR. Then the following are equivalent:
(i) The QIs of R is an idempotent.
(ii) P\(Q\S)⊆PΓQΓSwhenever P, Q, S are QIs of R ∋ P\(Q\S)≠ ∅.
(iii) $\langle a\rangle_q$ = Cube of [$\langle a\rangle_q$] = [$\langle a\rangle_q$]Γ[$\langle a\rangle_q$]Γ[$\langle a\rangle_q$]∀ *a*∈R.

Pf.: To show (I)⟹(II) Let P,Q and S are QIs of R such that P\ (Q\ S)≠ ∅. It is easy to show that P\ (Q\ S) is a QI of R. Since every QI of R is an idempotent, therefore P\ (Q\ S) = Cube of {P\ (Q\ S)}={P\ (Q\ S)}Γ{P\ (Q\ S)}Γ{P\ (Q\ S)}⊆PΓQΓS.
(II) ⟹(III) is obviously evident.
(III) ⟹(I) is plainly evident.

Def. 2.12: A set ∅ ≠X⊆ R, where *R* is a TGSR, is called an m_q-system if for every a, b, c ∈X∃ $a_1\in\langle a\rangle_q$, $b_1\in\langle b\rangle_q$ and $c_1\in\langle c\rangle_q$∋ $a_1\alpha b_1\beta c_1$∈X, for α, β∈ Γ.

Def. 2.13: A set ∅ ≠ X ⊆ R, where *R* is a TGSR, is said to bean n_q-system if for every *b* in X, ∃ b_1, b_2, $b_3\in\langle b\rangle_q$ such that $b_1\alpha b_2\beta b_3$∈ X, for α, β ∈ Γ.

Note 2.14: For every m_q-system ⟹ n_q-system. But, n_q-system ⇏ m_q-system.

Ex. 2.15: Let R = Γ = Z_6^- be the TGSR w.r.t. addition modulo 6 and multiplication modulo 6. Let X = $\{\overline{(-2)},\overline{(-3)}\}$. Then X is an n_q-system but not m_q-system.

Th. 2.16: Suppose R, a TGSR& Q, a QI of R. We prove the following
(i) B is a PQI⟺R \ B is an m_q-system.
(ii) B is a SPQI⟺R \ B is an n_q-system.

Pf.: (i) Suppose that B is a PQI of R. Let *a*, *b*, *c*∈R\ B. Let$a_1\alpha b_1\beta c_1$∈ R\ B,∀ $a_1\in\langle a\rangle_q$, $b_1\in\langle b\rangle_q$,$c_1\in\langle c\rangle_q$ and α, β∈ Γ ⇒$\langle a\rangle_q\Gamma\langle b\rangle_q\Gamma\langle c\rangle_q$⊆B.Since B is a PQI of R, ∴*a*∈B or *b*∈B or *c*∈B. It's wrong. Hence $a_1\alpha b_1\beta c_1$∈ *R*\ B for some $a_1\in\langle a\rangle_q$, $b_1\in\langle b\rangle_q$,$c_1\in\langle c\rangle_q$ and α, β ∈ Γ.Conversely, let P,Q and S be QIs of R such that PΓQΓS⊆ *B*. Assumethat P⊈B,Q⊈B and S⊈B. Let a ∈P\ B, b ∈Q\ B and c ∈S\ B. Then, *a*, *b*, *c*∈R\ B. Since R\ B is an m_q-system, therefore $a_1\alpha b_1\beta c_1$∈ R\B for some$a_1\in\langle a\rangle_q$, $b_1\in\langle b\rangle_q$,$c_1\in\langle c\rangle_q$ and α, β∈ Γ. But $a_1\alpha b_1\beta c_1\in\langle a\rangle_q\Gamma\langle b\rangle_q\Gamma\langle c\rangle_q$⊆ PΓQΓS⊆ B. This is false. Hence, P⊆ B or Q⊆ B or S ⊆B.Similarly, it is easy to prove(ii) also.

Def. 2.17: A QIB of a TGSRR is R-prime if xΓRΓyΓRΓz ⊆ B⟹x ∈ B or y ∈ B or z ∈ B. A QIB of a TGSRR is called R-semi-prime if xΓRΓxΓRΓx ⊆B⟹ x ∈B.

Th. 2.18: KΓMΓL ⊆B⟹K⊆B or M ⊆B or L ⊆B for any right ideal K, lateral ideal M and left ideal L of R⟺ B is R-prime when B is a QI of TGSR R.

Pf.: Let B be a R-prime QI of R and KΓMΓL ⊆B. Let us suppose K⊈B and M⊈B⟹ ∃ x ∈ *K*\B and y ∈M\B. Let z ∈L. Implies xΓRΓyΓRΓz ⊆KΓRΓMΓRΓL ⊆KΓMΓL ⊆B. Since B is R-prime we have x ∈B or y ∈B or z ∈B. x ∉ *B* and y ∉B⟹ z ∈B⟹ L ⊆B. Reversely, let us suppose xΓRΓyΓRΓz ⊆ *B*.Consider (xΓRΓR)Γ(RΓyΓR) Γ(RΓRΓy) ⊆xΓRΓyΓRΓz ⊆B. Since xΓRΓR is a right ideal, RΓyΓR is a lateral ideal and RΓRΓz is a left ideal, then by the given data, xΓRΓR⊆B or RΓyΓR⊆B or RΓRΓz ⊆B. If xΓRΓR⊆B, then xΓxΓx ∈xΓRΓR⊆B.Now$\langle x\rangle_r$Γ$\langle x\rangle_m$Γ$\langle x\rangle_l$ = (mx + xΓRΓR)Γ(mx + RΓxΓR + RΓRΓxΓRΓR)Γ(mx + RΓRΓx) ⊆ xΓxΓx + xΓRΓR⊆B.By the given data, $\langle x\rangle_r$⊆B or $\langle x\rangle_m$⊆ B or $\langle x\rangle_l$⊆ B. Thus, x ∈B. In the same way, if RΓyΓR⊆B ⟹ y ∈B and if RΓRΓz ⊆B⟹ z ∈B.If these cases are also similar a line saying the same to be mentioned. Hence B is R-prime.

Notation 2.19: LetB be a QI of a TGSR R. We have to define the following:
M(B) = {x ∈ B: RΓxΓR + RΓRΓxΓRΓR ⊆ B}
L(B) = {x ∈B: RΓRΓx ⊆ B}
R(B) = {x ∈B: xΓRΓR ⊆B}
I_L = {y ∈L(B): yΓRΓR⊆L(B)}
$_MI_M$ = {y ∈M(B): RΓyΓR + RΓRΓyΓRΓR⊆M(B)}
I_R = {y ∈R(B): RΓRΓy ⊆R(B)}

Prop. 2.20: Let B be a QI of a TGSRR. Then L(B)(resp. M(B),R(B)) is a left (resp. lateral, right) ideal of R⊆B if L(B)(resp. M(B),R(B)) is nonempty.

Pf.: Suppose b ∈L(B) anda$_1$, a$_2$∈ R. Then $a_1αa_2βb$ ∈ RΓRΓb ⊆ B. Now RΓRΓa_1Γa_2Γb ⊆ RΓRΓb ⊆ B. Thus, we have $a_1αa_2βb$ ∈L(B). Consequently RΓRΓL(B) ⊆L(B). So, L(B) is a left ideal of R. Using the same procedure, to prove that M(B) is a lateral ideal and K(B) is aright ideal of R.

Prop.2.21: AssumeB, a QI of a TGSRR. If ∅ ≠I_L (resp. $_RI$,$_MI_M$) then I_L (resp. $_RI$,$_MI_M$) is the largest ideal of R⊆B. Moreover I_L=$_RI$= $_MI_M$.

Pf.: Let b∈I_L. Then I_L⊆ L(B) ⊆B⟹b∈L(B) and b∈B. That is RΓRΓb⊆B. Then RΓRΓa_1Γa_2Γb ⊆ RΓRΓb ⊆ B for some a_1, a_2∈R⟹$a_1αa_2βb$ ∈ L(B). Since L(B) is a left ideal of R (Prop. 2.19) and bΓRΓR⊆L(B), we get$a_1αa_2βb$ΓRΓR ⊆ RΓRΓL(B) ⊆L(B). Thus $a_1αa_2βb$ ∈ I_L. That is RΓRΓI_L⊆I_L. Hence I_L is a left ideal of R. Similarly, we can show that I_L is a right ideal and a lateral ideal of R. Thus, I_L is an ideal of R⊆B.Let I be any ideal of R⊆B. Then RΓRΓI ⊆I ⊆B⟹ I ⊆L(B). Now IΓRΓR⊆I⊆L(B)⟹ I ⊆I_L. Hence I_L is the largest ideal of R⊆ Q. Using the same argument; we can prove that $_RI$ and $_MI_M$ are the largest ideals of R⊆ B. Since,I_L,$_RI$ and $_MI_M$ are the largest ideals of R⊆ B, therefore, I_L =$_RI$ =$_MI_M$.

Notation2.22: we denote I(B) =$_RI$ = I_L =$_MI_M$.

Prop. 2.23: Every QI is a PI of R, when B be a R-prime QI of a TGSRR.

Pf.: Given B is a R-prime QI of R. Suppose KΓMΓL ⊆I(B) for any ideals K, M, L of R. Now I(B) ⊆ L(B) ⊆ B⟹KΓMΓL ⊆B. Since B is R-prime, we have K⊆B or M ⊆B or L ⊆ B (by Theorem 2.19). Also, I(B)⊆B and is the greatest ideal ⟹ K ⊆ I(B) or M ⊆ I(B) or L ⊆ I(B). Hence, I(B)is a PI of R.

Cor. 2.24: I(B) is a semi-prime ideal of R, when B is a SPQI of a TGSR R.

Prop. 2.25: If B is R-semi-prime then B is a QI of R, when B be a Bi-ideal of a TGSR R.

Pf.: Let x ∈(BΓRΓR)∩(RΓBΓR+RΓRΓBΓRΓR)∩(RΓRΓB). Then x ∈ (BΓRΓR),x∈ (RΓBΓR+RΓRΓBΓRΓR) and x ∈(RΓRΓB).
Then, xαRβxγRδx ∈(BΓRΓR)ΓRΓ(RΓRΓBΓRΓR)ΓRΓ(RΓRΓB) ⊆ BΓRΓBΓRΓB ⊆B. Since B is R-semi-prime, we get x ∈B. Consequently, (BΓRΓR)∩(RΓBΓR+RΓRΓBΓRΓR)∩ (RΓRΓB) ⊆B. Hence B is a QI of R.

Prop. 2.27: If a TGSRR is regular, then every QI of R is R-semi prime.

Pf.: Suppose R is regular and B be a QI of R. Let aΓRΓaΓRΓa⊆B for a ∈R. Since R is regular, therefore for a ∈ R∃ x, y ∈ R, $α, β, γ, δ$ ∈ Γ such that $a = aαxβaγyδa$. Thus, $a = aαxβaγyδa$ ∈ aΓRΓaΓRΓa ⊆ B⟹a∈B. Hence B is R-semi prime.

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