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Semi-analytical Evaluation of Entropy Generation in 1-D Heat Conduction with Variable Thermal Conductivity

T. Anupkumar[a,c], A. Praveen Reddy[a], Noble Sharma[a], N. Narayan Rao[b] and B. Srinivasa Rao[b]

[a]*Dept. of Mech. Engg., Koneru Lakshmaiah Education Foundation, Guntur, India*
[b]*Dept. of Mech. Engg., Vignan's Foundation for Sci., Tech. & Research (Deemed to be University), Guntur, India*
[c]*Corresponding Author, Email: anupkumar@kluniversity.in*

ABSTRACT:

In the present study, steady state heat transfer in a slab is analysed by applying the principle of variation calculus to the entropy generation minimization. The governing equation of the phenomena is obtained by minimizing the total entropy generation over the slab by considering the irreversibility and variation of thermal conductivity as a function of spatial co-ordinates. The governing equation is solved to obtain the temperature distribution, internal heat generation due to irreversibility, entropy generation number and entropy transport into system. The apparent heat sources that come into existence because of the irreversibility in heat diffusion have made the minimization of entropy generation feasible.

KEYWORDS:

Entropy generation minimization; Irreversibility; Internal heat sources; Euler-Lagrange

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NOMENCLATURE:

k	Thermal conductivity
$\dot{q}$	Thermal flux (W/m^2)
$\dot{q}_v$	Volumetric intensity of internal heat source (W/m^3)
T	Temperature (K)
x_i	Cartesian co-ordinates (i=1,2,3)
$\sigma_t, \sigma_{t,min}$	Global and minimized global intensity of entropy generation rate (W/K)
θ	Transformed temperature
Ω	Domain of interest
σ_{ext}	Entropy transport

1. Introduction

Steady thermal diffusion phenomenon which is derived based on the application of energy conservation principle over a differential control volume is described by,

$$\nabla^2 T + \frac{q_v}{K} = 0 \quad (1)$$

Explicit heat sources within the domain, if exists, are part of (q_v) in this governing equation. In the absence of internal heat sources, the phenomena are represented by,

$$\nabla^2 T = 0 \quad (2)$$

Taking the reality of the existence of irreversibility into consideration there should be an alternate governing equation to represent the phenomena of heat transfer by conduction. However, these equations of irreversible heat transfer need to be derived of the entropy generation and must be expressed in terms of temperature and its derivatives. Thus, to get a differential equation which describes the heat conduction phenomena that is more realistic by considering the irreversibility that occur in heat transfer as well as being pragmatic by considering the phenomena to be quasi equilibrium, an alternative approach of "Entropy generation minimization" technique is proposed [1-3]. In this technique, variation calculus is utilised to derive the governing differential equation by minimizing a functional with respective to temperature as referred as "Entropy generation" [4].

A classical boundary value problem in heat conduction has been considered and entropy generation technique was applied to derive the governing differential equations for steady state heat conduction problem with internal heat generation using,

$$div[k(x) grad T(x_i)] + \dot{q_v}(x_i) = 0 \quad (3)$$

This is obtained by invoking the first law of thermodynamics over a differential volume in absence of work transfer [5-8]. The alternate formulation for the same equation is obtained by utilizing the entropy generation minimization principle. The total entropy generation integrating the local entropy generation is given by Sciacovelli [9] as,

$$\sigma_t = \sigma\left(T, S_{x_i}\right) \cong \frac{d_i\sigma}{dT} = \frac{k(T)}{T^2}\left(\frac{\partial T}{\partial x_i}\right)^2 \quad (4)$$

Where T and Tx_i are the temperature field and gradient of temperature field with respect to the space co-ordinates x_i. Then the alternate formulation is cast by finding a suitable function Tx_i that satisfies the boundary conditions of a given problem and also minimizes the global entropy generation function (i = 1,2,3) given by,

$$\sigma_t = \int_0^\Omega \sigma\left(T, T_{x_i}\right) d\Omega \quad (5)$$

The desired temperature T(x) simultaneously satisfies the boundary conditions and minimizes the σ_t as given by the Euler-Lagrange equation as,

$$\frac{\partial \sigma}{\partial T} - \sum_i \frac{\partial}{\partial T_{x_i}}\left(\frac{\partial \sigma}{\partial T_{x_i}}\right) = 0 \quad (6)$$