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Study of Heat and Mass Transfer in a Rotating Nanofluid Layer Under Gravity Modulation

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In this paper we investigate the effect of gravity modulation and rotation on thermal instability in a horizontal layer of a nanofluid. Finite amplitudes have been derived using the minimal Fourier series expressions of physical variables in the presence of modulation and slow time. Here we incorporates the layer of nanofluid with effect of Brownian motion along with thermophoresis. Heat and mass transfer are evaluated in terms of finite amplitudes and calculated by Nusselt numbers for fluid and concentration. It is found that, gravity modulation and rotation can be used effectively to regulate heat and mass transfer. This modulation can be easily felt by shaking the layer vertically with sinusoidal manner. The numerical results are obtained for amplitude of modulation and presented graphically. It is found that rotation and frequency of modulation delays the rate of heat and mass transfer. This shows that a stabilizing nature of gravity modulation and rotation against a non rotating system. A comparison made between modulated and unmodulated and found that modulated system influence the stability problem than un modulated system. Similarly modulated system transfer more heat mass transfer than unmodulated case. Finally we have drawn streamlines and nanoparticle isotherms to show the convective phenomenon.

KEYWORDS: Nanofluids, Rotation, Nonlinear Model, Gravity Modulation, Minimal Fourier Series.

1. Introduction

Studies related to fluid engineering, lubricants, manufacture of nano-meter sized particles and micro-meter sized particles is very important to enhance system performance and thermal properties. Especially material formation, fiber formation, oil, machine performance these fluids are play an important role. The usage of nanofluid was suggested by Maxwell[1] earlier. The smaller size particles provides larger relative surface area than microsized particles improving the heat/mass transfer properties. So these fluids may overcome the problems related to clogging channels, producing drastic pressure drops, settling, and premature wear on channels and components. Enhanced heat transfer may reduce pumping power, and minimal clogging, innovation of miniaturized systems leading to savings of energy and cost. Choi[2] explained potential applications and benefits of nanofluids in terms of production, characterization, and performance of nanofluids.

The convective instability with time-dependent gravity modulation is very important because of their application in engineering and science. The fluctuations in gravity field considered in the buoyancy driven convection may be used as a tool to increase the heat and mass transport, hence affecting its stability problem. Gershuni et al.[3] and Gresho and Sani[4] were the first to study the influence of gravity modulation in stability analysis. We know that nanofluids are fluids formed by suspensions of nanometer-sized particles in base fluids. One may observe that, once nanofluids suspended in base fluids all the fluid properties will change. In this case we need to understand flow mode to control instability. We observe that detailed applications of nanofluid has been given by Wong and Leon[5] The basic idea of nanofluid convection was introduced in the studies of Refs. [6–8]. The study of nanofluid rotating porous convection was investigated by Agarwal et al.[9] Buongiorno[10] investigated nanofluid boundary layer flow. It is found that nanofluid properties significantly vary within the boundary layer because of the effect of temperature gradient and thermophoresis. Also these effects can result in a significant decrease of viscosity within the boundary layer, thus leading to heat transfer enhancement. A series of works on nanofluids while employing the stability criteria was well documented by Nield and Kuznetsov.[11–12] The study of steady, laminar, mixed convection boundary-layer flow

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over a vertical cone embedded in a porous medium saturated with a nanofluid is studied by Chamkha et al.[13] Applying the different models for evaluating radiative flow Ref. [40] of sutterby nanofluid with variable zt was reported by Sohail and Nazir.[14]

The reader may see other studies on nanofluids which tells the importance of nanofluid applications. The study of unsteady mixed convection of nanofluids has been investigated by Takhar et al.,[15] Chamkha et al.[16,17] without any modulation. The effect of hall and ion slip on rotating boundary layer flow of nanofluid past an infinite vertical plate embedded in a porous medium has been investigated by Krishna and Chamkha.[18] Hall effects on unsteady MHD oscillatory free convective flow of second grade fluid through porous media was given by Krishna et al.[19,20] The effect of entropy generation on nanofluid convection has been studied by Parvin and Chamkha[21] and Muneer et al.[22] The effect of throughflow on weakly nonlinear convection in a viscoelastic porous medium was given by Kiran et al.[23] The effect of thermal radiation and shape factor of nanoparticles impacts on nanofluid convection was given by Chamkha et al.[24] Other related studies of nanofluid convection are presented in the following studies.[25–27] The studies related to double diffusive convection or heat mass transfer for different porous layers recently was given by Krishna et al.[28–30] and Eshaghi et al.[31] The effect of gravity modulation on nonlinear thermal instability of Rayleigh Benard convection was reported by Bhadauria and Kiran.[32–34] The study of heat mass transfer in layer of nanofluis with mhd has been reported by Krishna et al.[35–37] and Amine et al.[38] and Chamkha et al.[39]

Rayleigh–Benard convection in a nanofluid layer using a thermal nonequilibrium model was given by Agarwal et al.[41] Convection in a double diffusive nanofluid saturated rotating porous layer was given by Puneet and Agarwal.[42] The effect of gravity modulation on thermal convection in a nanofluid saturated porous medium was given by Kiran et al.[43] Centrifugally driven convection in a nanofluid saturated rotating porous medium with rotation speed modulation was given by Kiran and Narasimhulu.[44] Other studies of modulation on nanofluid convection was discussed and well documented in Refs. [45–47]. Agarwal[56] investigated natural convective flow of nanofluid-saturated rotating porous media with more realistic boundary conditions. In continuation with,[56] the flow patterns in linear state of Rayleigh Benard convection in a rotating nanofluid layer presented in Ref. [57]. The effect of thermal modulation on the onset of convection in a porous layer was given by Umavathi.[58] The study of nonlinear stability analysis of rotating porous media with thermal non-equilibrium approach utilizing the fluids Al_2O_3-EG Oldroyd-B nanofluid investigated by Agarwal and Rana.[59] Analysis of unsteady heat and mass transfer in a rotating nanofluid layer was given by Agarwal and Bhadauria.[48] Krishna and Chamkha[49,50] explained mhd flow of nanofluids in the presence of nanofluids saturating porous media. Other studies on nanofluid convection for different models are reported by Daneshfar et al.,[51] Maleki et al.[52] and Krishna et al.[53,54]

The novelty of the present work is to study the time dependent gravity modulation on rotating machineries i.e., rotating porous media. For example (rotating porous layer applications) applications like biomechanics, petroleum extraction industries, nuclear reactors, and geophysical problems, correspond to the classical rotation problem of porous media. We have seen numerous research articles without modulation for nonlinear studies. It is very much important to control transport phenomenon in terms of stability and nonlinear analysis. However, to the best of author's knowledge till date no study reports the effect of gravity modulation in the presence of rotational effects on nanofluid convection. The literature shows that nanofluid convection for linear theories and discussed only stability criteria. But, till date no study investigated gravity modulation in a rotating nanofluid layer. Therefore, the purpose of the present study is to see the effect of gravity modulation on rotating nanofluid layer and quantify heat and concentration transport.

2. MATHEMATICAL FORMULATION

A layer of nanofluid confined between 2 free–free horizontal infinitely extended and located at $z = 0$ and $z = d$, has been consider. The Rayleigh Benard analysis is consider in the layer which heated from below and cooled from above. The layer is assumed to rotate uniformly about vertical axis z with uniform angular velocity $\vec{\Omega}$. For rotational effects we have altered the momentum equation by adding the term of Coriolis force. The gravitational force assumed to be periodic with finite amplitudes. The oscillations of gravity are considered to be low frequency in order to avoid form continuous convective rolls. The reader may note that (from the Fig. 1) the nanofluid layer is extended infinitely in direction of x and y and z axis is taken vertically upward. Here T_h and T_c are the temperatures at the lower and upper walls respectively such that $T_h > T_c$ since lower plate always hotter than upper one. The governing equations of the problem (with Oberbeck-Boussinesq approximation) are given by (see for reference Tzou,[6,7] Nield and Kuznetsov[8], Agarwal et al.[9] and Buongiorno[10]).

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\rho_f \frac{\partial \tilde{\boldsymbol{q}}}{\partial t} + \rho_f(\vec{q} \cdot \nabla \vec{q}) + \nabla p = \mu \nabla^2 \vec{q} + [\phi \rho_p + (1-\phi) \times \{\rho(1-\beta(T-T_c))\}]\vec{g}(t) + \frac{2}{\varepsilon}(\vec{q} \times \Omega) \tag{2}$$

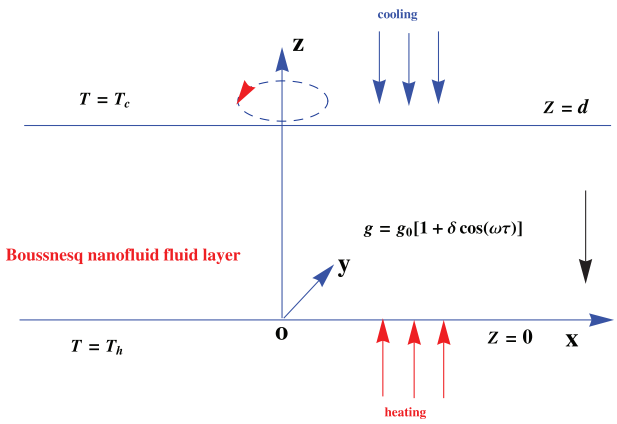

Fig. 1. Graphical interpretation of the problem.

$$\frac{\partial T}{\partial t}+\vec{q}\cdot\nabla T=\kappa_T\nabla^2 T+\frac{(\rho c)_p}{(\rho c)_f}\times\left[D_B\nabla\phi\cdot\nabla T+\frac{D_T}{T_c}\nabla T\cdot\nabla T\right] \tag{3}$$

$$\frac{\partial \phi}{\partial t}+\vec{q}\cdot\nabla\phi=D_B\nabla^2\phi+\frac{D_T}{T_c}\nabla^2 T \tag{4}$$

where $\vec{q}$ is the fluid velocity. The physical variables have their usual meanings, given in nomenclature. The time dependent gravity modulation terms has been considered in this paper as

$$\vec{g}(t)=g_0\left(1+\delta\cos(\omega t)\right)\vec{k} \tag{5}$$

where δ, ω is the modulation amplitude and modulation frequency. Here after we write $(1+\delta\cos(\omega t)$ as Gm. We consider temperature and volumetric fraction of the nanoparticles to be constant at the boundaries which are stress-free. In this case we may assume the boundary conditions on T and ϕ to be:

$$v=0, \quad T=T_h, \quad \phi=\phi_1 \quad \text{at } z=0 \tag{6}$$

$$v=0, \quad T=T_c, \quad \phi=\phi_0 \quad \text{at } z=d \tag{7}$$

The dimensionless variables are considered as given below: $(x^*, y^*, z^*)=(x, y, z)/d$, $t^*=t\kappa_T/d^2$, $(u^*, v^*, w^*)=(u, v, w)d/\kappa_T$, $p^*=pd^2/\mu\kappa_T, \phi^*=(\phi-\phi_0/\phi_1-\phi_0)$ and $T^*=(T-T_c/T_h-T_c)$, where $\kappa_T=(k_m/(\rho c)_f)$. The non-dimensionalized governing Eqs. (1)–(6) along with boundary conditions are (after dropping the asterisk for simplicity):

$$\nabla\cdot\vec{q}=0 \tag{8}$$

$$\frac{1}{Pr}\frac{\partial\vec{q}}{\partial\tau}+\frac{1}{Pr}\vec{q}\cdot\nabla^2\vec{q}=-\nabla p-\nabla^2\vec{q}-(Rm-Ra\ T+Rn\phi)\times Gm\hat{e}_z+\sqrt{Ta}(\vec{q}\times\hat{k}) \tag{9}$$

$$\frac{\partial T}{\partial t}+\vec{q}\cdot\nabla T=\nabla^2 T+\frac{N_B}{Le}\nabla\phi\cdot\nabla T+\frac{N_A N_B}{Le}\nabla T\cdot\nabla T \tag{10}$$

$$\frac{\partial\phi}{\partial t}+\vec{q}\cdot\nabla\phi=\frac{1}{Le}\nabla^2\phi+\frac{N_A}{Le}\nabla^2 T \tag{11}$$

The boundary conditions given in Eq. (7) will take the form:

$$\vec{q}=0, \quad T=1, \quad \phi=1 \quad \text{at } z=0 \tag{12}$$

$$\vec{q}=0, \quad T=0, \quad \phi=0 \quad \text{at } z=d \tag{13}$$

The non-dimensionlized parameters in the above equations are given in Nomenclature, N_A is the modified diffusivity ratio, which is similar to the Soret parameter that arises in cross diffusion in thermal instability. When the fluid is at rest then the heat and mass transfer will be of conduction form. At this state, the physical quantities are of the form:

$$\vec{q}=0, \quad p=p_b(z), \quad T=T_b(z), \quad \phi=\phi_b(z) \tag{14}$$

It is noted that basic state terms are of function of z variable. At initial state heat and mass transfer in the form of conduction near to the walls. Substituting the Eq. (14) in Eq. (8) and Eq. (11), we get:

$$\frac{d^2T_b}{dz^2}+\frac{N_B}{Le}\frac{d\phi_b}{dz}\frac{dT_b}{dz}+\frac{N_A N_B}{Le}\left(\frac{dT_b}{dz}\right)^2=0 \tag{15}$$

$$\frac{d^2\phi_b}{dz^2}+N_A\frac{d^2T_b}{dz^2}=0 \tag{16}$$

while employing an order of magnitude analysis,[9,42,43,57,18] we have:

$$\left(\frac{d^2T_b}{dz^2}, \frac{d^2\phi_b}{dz^2}\right)=(0, 0) \tag{17}$$

The solutions of the Eq. (17) subject to the boundary conditions Eq. (13) are:

$$(T_b(z), \phi_b(z))=(1-z, 1-z) \tag{18}$$

We now disturb the system to study convection analysis. We superimpose perturbations on the basic state given in Eq. (14):

$$\vec{q}=\vec{q}', \quad p=p_b+p', \quad T=T_b+T', \quad \phi=\phi_b+\phi' \tag{19}$$

Substituting the Eq. (19) in Eqs. (8)–(11) and using the expressions (18) and (19), eliminating the pressure and introducing the stream functions $u=(\partial\psi/\partial z)$; $v=-(\partial\psi/\partial x)$, by taking the curl of the momentum equation twice, the system of nonlinear perturbed equations are:

$$\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2\psi-\nabla^4\psi-Ta\frac{\partial^2}{\partial z^2}\frac{\partial\psi}{\partial x}=-RaGm\frac{\partial T}{\partial x}+RnGm\frac{\partial\phi}{\partial x}+\frac{\partial(\psi, \nabla^2\psi)}{\partial(x, z)} \tag{20}$$

$$\frac{\partial T}{\partial t}+\frac{\partial \psi}{\partial x}=\nabla^2 T+\frac{\partial(\psi, T)}{\partial(x, z)} \tag{21}$$

$$\frac{\partial \phi}{\partial t}+\frac{\partial \psi}{\partial x}=\frac{1}{Le}\nabla^2\phi+\frac{N_A}{Le}\nabla^2 T+\frac{\partial(\psi, \phi)}{\partial(x, z)} \tag{22}$$

The following boundary conditions are consider here (Nield, and Kuznetsov,[8] Agarwal et al.,[9] Buongiorno,[10] Bhaduria and Agarwal,[62] Kiran and Bhaduria[71]):

$$\psi=\frac{\partial^2\psi}{\partial z^2}=0, \quad T=0, \quad \phi=0 \quad \text{at } z=0 \text{ and } z=1 \tag{23}$$

3. NONLINEAR STABILITY ANALYSIS

Assuming Fourier series expressions (Agarwal et al.,[9] Bhadauria and Agarwal[62]) for the stream function, temperature and nanoparticle fractions, a local nonlinear stability analysis will be performed. We consider the modes (1,1) for stream function, (0,2) and (1,1) for temperature and nanoparticle concentrations, resulting in the following truncated representation of Fourier series (Veronis,[60] Rudraiah.[61])

$$\psi=L_{11}(\tau)\sin(\alpha x)\sin(\pi z) \tag{24}$$

$$T=M_{11}(\tau)\cos(\alpha x)\sin(\pi z)+M_{02}(\tau)\sin(2\pi z) \tag{25}$$

$$\phi=N_{11}(\tau)\cos(\alpha x)\sin(\pi z)+N_{02}(\tau)\sin(2\pi z) \tag{26}$$

where the amplitudes $L_{11}(\tau)$, $M_{11}(\tau)$, $M_{02}(\tau)$, $N_{11}(\tau)$, $N_{02}(\tau)$ and $O_{11}(\tau)$ are functions of time and to be determined. Substituting the Eqs. (24)–(26) in Eqs. (20)–(22), taking the orthogonality condition with the eigenfunctions associated with the considered minimal mode, we get:

$$\frac{dL_{11}(\tau)}{d\tau}=O_{11}(\tau) \tag{27}$$

$$\begin{aligned}\frac{\alpha c}{Pr^2}\frac{dO_{11}(\tau)}{d\tau}&=L_{11}(\tau)\left(\frac{c\alpha}{Pr}(Ra-Rn)Gm-Ta\pi^2\alpha-c^3\alpha\right)\\&+M_{11}(\tau)Gm\left(Rac^2(Pr^{-1}-1)-\frac{RnNac^2}{PrLe}\right)\\&+L_{11}(\tau)M_{02}(\tau)\frac{\pi\alpha cRa}{Pr}Gm\\&+N_{11}(\tau)(Rnc^2(1-(LePr)^{-1}))Gm\\&-L_{11}(\tau)N_{02}(\tau)\frac{\pi\alpha Rnc}{Pr}Gm-\frac{2\alpha c^2}{Pr}O_{11}(\tau)\end{aligned} \tag{28}$$

$$\frac{dM_{11}(\tau)}{d\tau}=-[\alpha L_{11}(\tau)+cM_{11}(\tau)+\pi\alpha L_{11}(\tau)M_{02}(\tau)] \tag{29}$$

$$\frac{dM_{02}(\tau)}{d\tau}=\frac{\alpha\pi}{2}L_{11}(\tau)M_{11}(\tau)-4\pi^2M_{02}(\tau) \tag{30}$$

$$\frac{dN_{11}(\tau)}{d\tau}=-\left[\alpha L_{11}(\tau)+\pi\alpha L_{11}(\tau)N_{02}(\tau)-\frac{c}{Le}[NaM_{11}(\tau)+N_{11}(\tau)]\right] \tag{31}$$

$$\frac{dN_{02}(\tau)}{d\tau}=\frac{\pi\alpha}{2}L_{11}(\tau)N_{11}(\tau)-\frac{4\pi^2}{Le}[NaM_{02}(\tau)+N_{02}(\tau)] \tag{32}$$

The above system of simultaneous autonomous ordinary differential equations can be subsequently solved numerically using Mathematica ND Solve.

3.1. Transport Phenomenon

The thermal Nusselt number, $Nu(\tau)$ is defined as

$$Nu=\frac{\text{Heat transport by (conduction+convection)}}{\text{Heat transport by conduction}}=1+\left[\frac{\int_0^{2\pi/\alpha_c}((\partial T\partial z))\,dx}{\int_0^{2\pi/\alpha_c}(\partial T_b/\partial z)\,dx}\right]_{z=0} \tag{33}$$

Substituting the Eqs. (24) and (25) in Eq. (33), we get

$$Nu(\tau)=1-2\pi M_{02}(\tau) \tag{34}$$

The nanoparticle concentration Nusselt number, $Nu_\phi(\tau)$ is defined by.

$$Nu_\phi=\frac{\text{Heat transport by (conduction+convection)}}{\text{Heat transport by conduction}}=1+\left[\frac{\int_0^{2\pi/\alpha_c}\left(\frac{1}{Le}\frac{\partial\phi}{\partial z}+\frac{N_A}{Le}\frac{\partial T}{\partial z}\right)dx}{\int_0^{2\pi/\alpha_c}\left(\frac{1}{Le}\frac{\partial\phi_b}{\partial z}+\frac{N_A}{Le}\frac{\partial T_b}{\partial z}\right)dx}\right]_{z=0} \tag{35}$$

$$Nu_\phi(\tau)=Na[1-2\pi M_{02}(\tau)]+[1-2\pi N_{02}(\tau)] \tag{36}$$

The system of Eqs. (27)–(32) are dissipative and bounded. One may also conclude that trajectories of this system will be confined to the finiteness of ellipsoid.

4. RESULTS ANALYSIS

We have investigated nonlinear convection in a rotating nanofluid layer under gravity modulation. The nonlinear system of equations are solved numerically and found heat/mass transfer quotients Nu and Nu_ϕ. Low amplitude of modulation has been considered i.e., $\delta=0.1$ due to finite amplitude convection. The system Eqs. (27)–(32) are solved with suitable initial conditions near 0.2. One may observe that the figures oscillate vigorously initially but attain a constant value as time passes by.

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Which indicates that, unsteady rate of heat/mass transfer initially which changes to a constant rate with the passage of time. The oscillations varies with respect to modulation amplitude. The parameters of the present problem are $Pr, Ta, Le, Na, \delta, \omega$ and they have chosen the fixed values 1.0, 10, 40, 2, 0.1 and 6. To see the effect of different parameters on heat/mass transfer. Further, we discussed the nature of streamlines and isotherms with respect to slow time. The graphical representation of the results are presented in Figures 1–7.

The effect of various nanofluid and modulation parameters have been obtained and depicted graphically. We have drawn attention to the heat/mass transfer analysis by nonlinear theory. Now let us discuss the results related to nonlinear system. The heat transfer results are presented in Figures 2 and 3 and concentration transport in Figures 4 and 5. In Figure 2 Nu starts with 1 indicating that the conduction state for lower values of time, on further values of time, Nu grows for convection in progress. We have presented the results corresponding to Pr on Nu and Nu_ϕ in Figures 2(1) and 4(1). Its effect has been presented in the Figure 2(1). From the figure, we see that initially there is no variation in Nu and Nu_ϕ. For later there is enhancement in Nu and Nu_ϕ as the value of PrantdL number Pr increases. Thus, showing the heat and concentration transport increases when Pr increases. But for large values of τ, the disturbances become oscillatory and the values of Nu and Nu_ϕ approach steady mode. The reader may observe that the value of Pr can be taken more than one, in that case the effect of local acceleration term will not affect the convection problem. For references related to Pr, the reader may observe few studies which are given by Agarwal et al.,[9,62] Kiran and Narasimhulu[44] and Yadav et al.[68] Reader may point that the same results may be obtained for Nu_ϕ given in Figure 4(1).

The effect of nanoparticle concentration Rn on Nu and Nu_ϕ is presented in Figures 2(2) and 4(2). It is observed that for $Rn > 0$ the suspended particle increases as a consequence heat/mass transfer increases, and an opposite case i.e., when $Rn < 0$ there is decrement in Nu and Nu_ϕ. The above corresponding results on heat transfer may be compared with the studies of Krishna et al.,[69,70] Agarwal et al.[41] and Puneet and Agarwal[42] The effect of modified diffusivity ratio Na is to increases the value of rate of mass transfer as evident from Figure 4(3). But no significant effect observed on Nu (in Fig. 2(3)) because Na is related nanofluids. The similar results for Le also can be seen. What we have understood here Na and Le shows their effect on Na only but not on Nu. These are the results reported earlier by Kiran et al.[43,44] and Bhadauria and Agarwal.[62]

The effect of rotation in terms of Taylor number (Ta) is presented in Figures 3(1) and 5(1) and found reduction in heat/mass transfer. This behaviour of Ta is

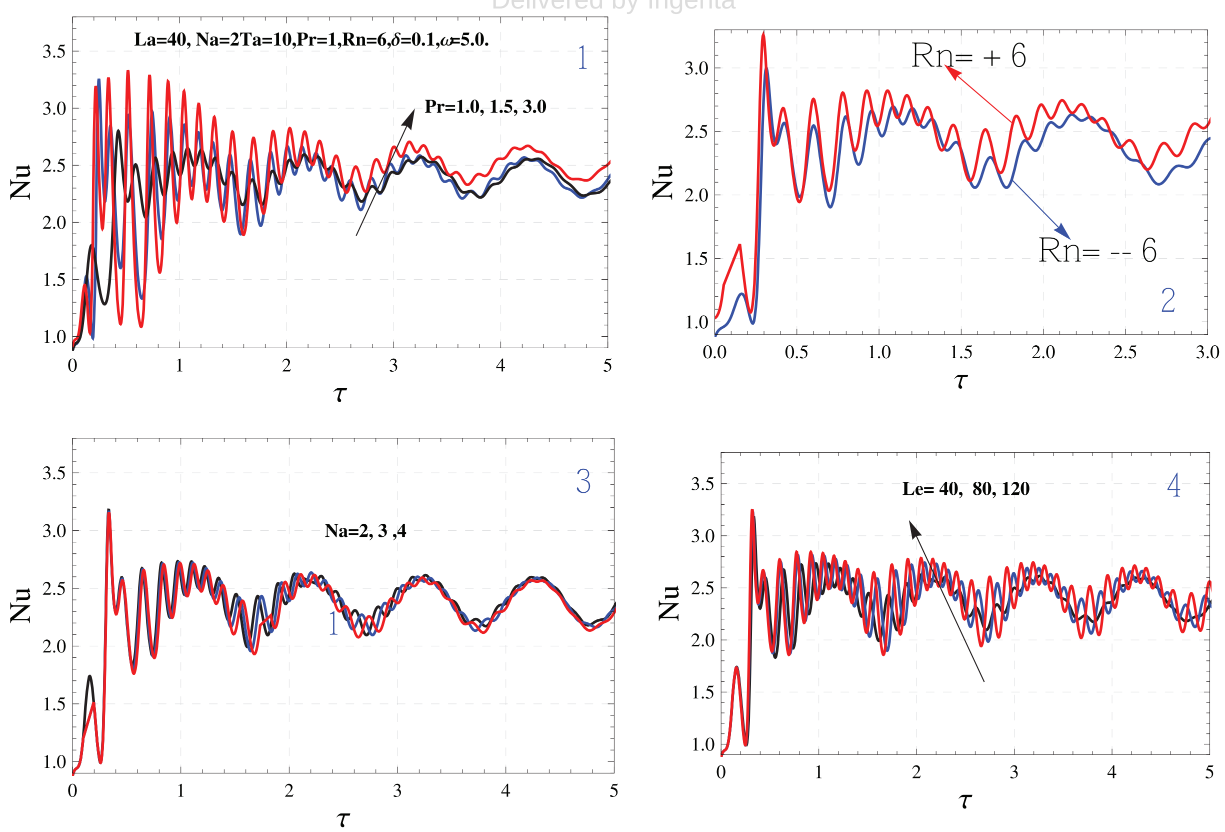

Fig. 2. Heat transfer results for (1) Pr (2) Rn (3) Na (4) Le.

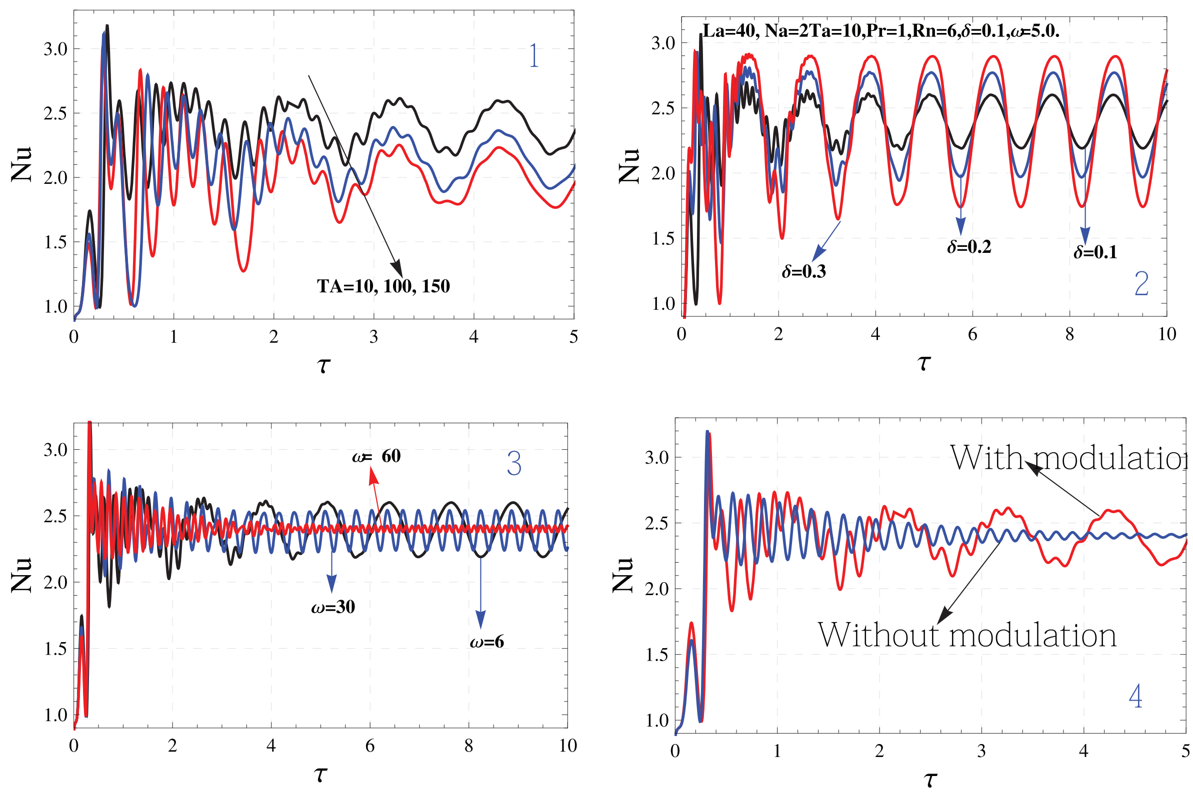

Fig. 3. Heat transfer results for (1) Ta (2) δ (3) ω (4) comparison.

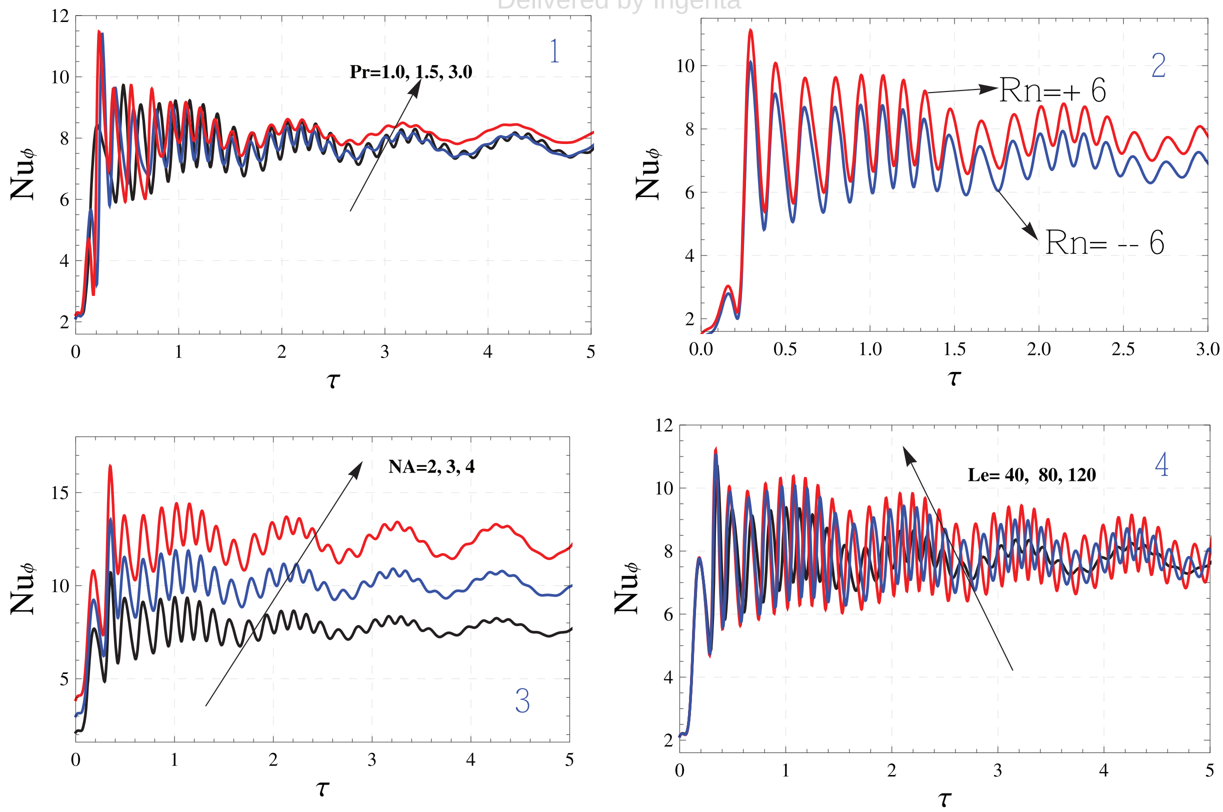

Fig. 4. Heat transfer results for (1) *Pr* (2) Rn (3) Na (4) Le.

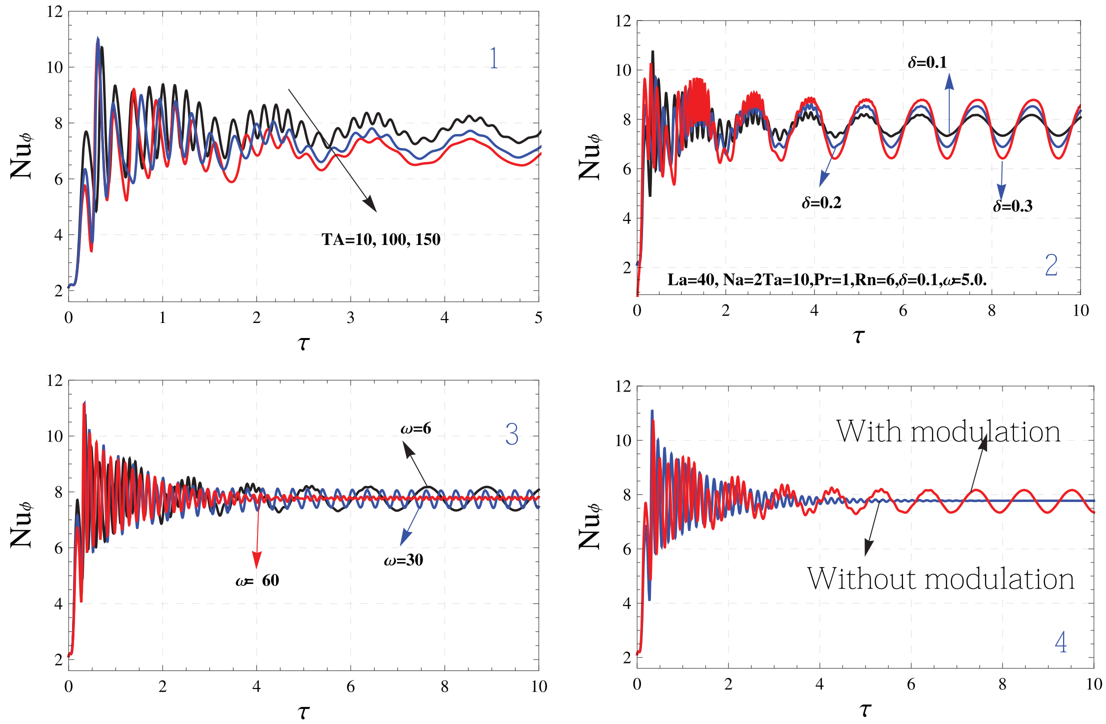

Fig. 5. Heat transfer results for (1) Ta (2) δ (3) ω (4) comparison.

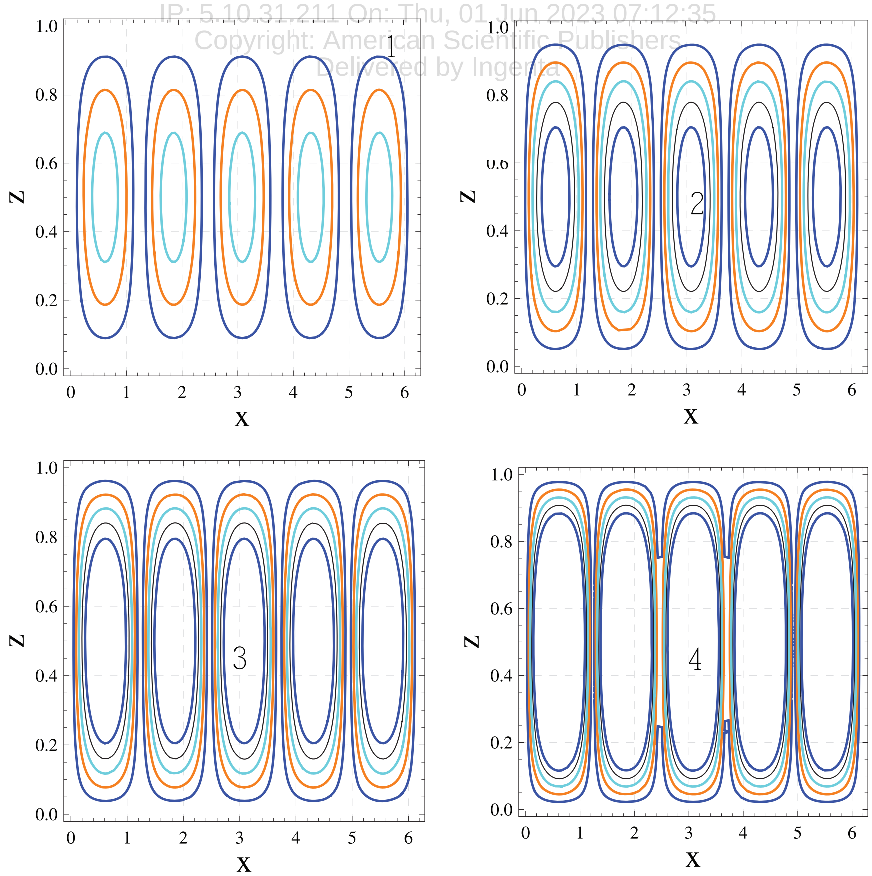

Fig. 6. Continued.

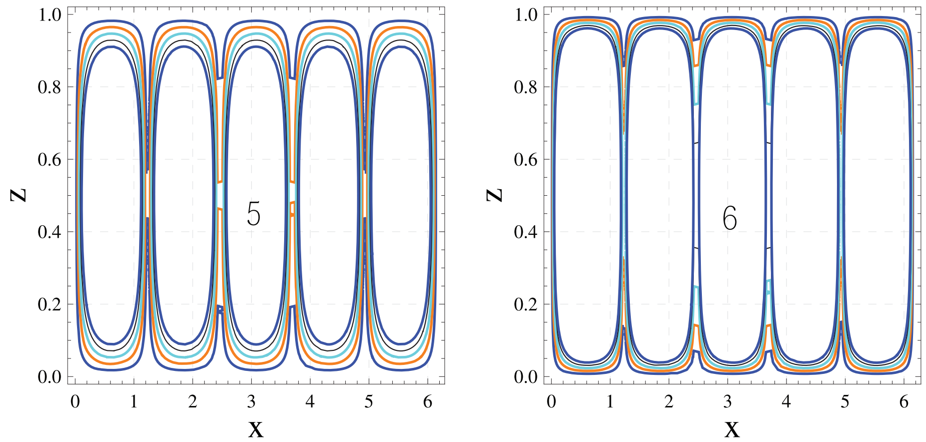

Fig. 6. Streamlines at different instances (1) $\tau=0$ (2) $\tau=0.5$ (3) $\tau=1$ (4) $\tau=2$ (5) $\tau=3$ (6) $\tau=5$.

common in nature and conforms the result of (Donnelly,[63] Bhattacharjee,[64] Om et al.[65,66] and Bhadauria and Kiran[67]), for nonlinear models. Onset criteria may vary depending on low and higher values of ω. The dual nature stability or instability of the flow may be adjusted with proper tuning of Ta and ω. The amplitude of modulation is given in Figures 3(2) and 5(2), and found heat/mass transfer enhancement. In Figures 3(2) and 5(2) the vibrations becomes high as the value of amplitude increases, and so thermal Nusselt (Nu) number and nanoparticle concentration Nu_ϕ increase, thus increasing the rate of heat/mass transport. These are the results reported by Bhadauria and Kiran,[32–34] Bhadauria Kiran[46] and Kiran.[47]

Let us now discuss the effect of frequency of modulation on heat/nanoparticle concentration transfer. It is clear

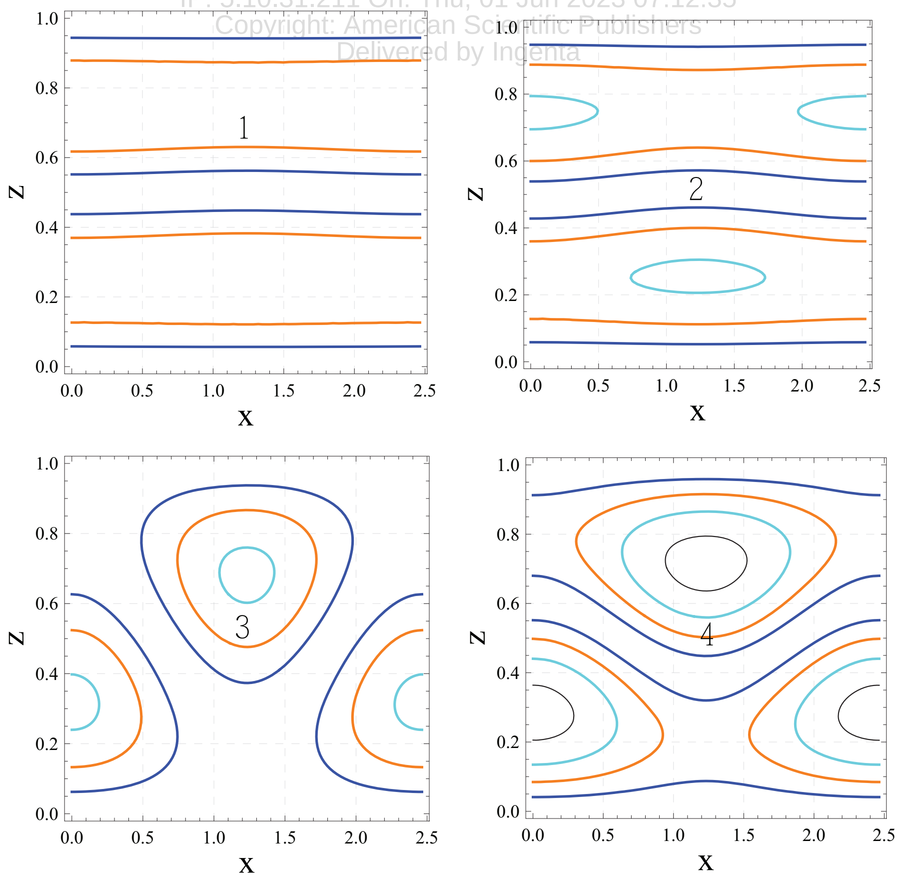

Fig. 7. Continued.

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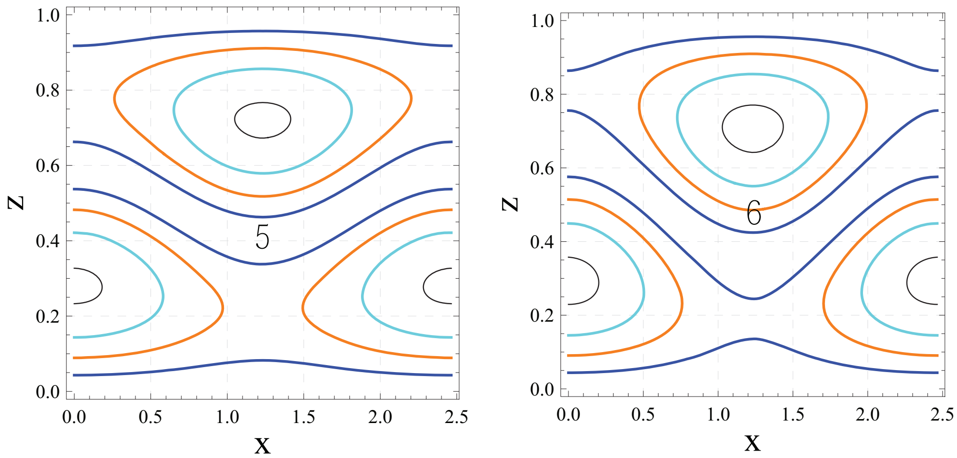

Fig. 7. Isotherms at different instances (1) $\tau = 0$ (2) $\tau = 0.5$ (3) $\tau = 1$ (4) $\tau = 2$ (5) $\tau = 3$ (6) $\tau = 5$.

from the Figures 3(3) and 5(3) that, for lower values of ω there is significant fluctuations in Nu and Nu_ϕ but, for larger values of ω there is no variation on Nu and Nu_ϕ thus, modulation disappears. These results conforms the linear theory results. For heat transfer results the reader may look at the studies of Kiran,[47] Bhaduria et al.,[67,70] and Kiran and Bhadauria[71] and Kiran.[72] The nature of gravity modulation is clear and indicates that, amplitude of modulation enhances and frequency of modulation diminishes the heat/mass transfer in the system. The effect of rotation also may be observed and found reduction in heat transfer.

In Figures 6 and 7 we have depicted streamlines and isotherms to see the nature of convective phenomenon. As slow time varies from $\tau = 0.0$ to $\tau = 4$ there is significant effect of convection can be observed. The time-dependent terms of ψ (streamlines), T (isotherms), as a function slow time we have drawn the figures. Here we are not presenting the results of ϕ (isohalines), at different times due to repetition. It is clear that with increasing in time, magnitudes of streamlines, and isotherms are varying from their initial state Figures (6(1), 6(2), 7(1), 7(2)). In streamlines one can observe enhancement in cell size, similarly in the case of isotherms, isotherms are more vibrant as time goes on Figures (6(3), 6(4), 7(3), 7(4)). In all the figures there will be a rigorous change in streamlines and isotherms. In the case of isotherms, at initial state convection in the form of conduction which approaches to convection later state. The magnitude of isotherms increasing with slow time and achieves its un changeable state after $\tau = 4$ see in Figures (6(5), 6(6), 7(5),7(6)). It is quite natural to see the nature of streamlines and isotherms for Benard convection. The same results we obtained for the current problem. Our results are quite agree with the results obtained previously by Bhadauria and Kiran,[32–34] Kiran et al.,[43] Kiran,[47] and Manjula et al.[73–77] In the intermediate time range, uniform convection cells are observed which change to strong convection with the passage of time.

5. CONCLUSIONS

We have investigated the effect of gravity modulation on nanofluid convection with rotation, which is heated from below and cooled from above. Incorporating the effect of Brownian motion along with thermophoresis the nonlinear theories are developed. Based on our study the following conclusions are drawn:

The effect of gravity modulation has dual role on heat and mass transport. It is found that low frequency of modulation shows destabilize effect but, high frequency modulation stabilize the system. The effect of rotation is to decrease (stabilizes) the heat and nanoparticle concentration. Increment in concentration Rayleigh number Rn, modified diffusivity ratio N_A and Lewis number Le increases the effect of modulation on Nu_ϕ. An increment in Pr, δ enhances the heat and concentration transport. It is found that no significant effect of N_A and Le on Nu. The nature of Rn has dual role i.e., positive values of Rn, enhances Nu and Nu_ϕ and negative values of Rn shows reduction on Nu and Nu_ϕ. Further, it is found that Le, N_A and Rn is to enhance concentration transport, whereas ω decreases the heat and concentration transport. Finally the modulation can be used to regulate transport phenomenon in the system.

NOMENCLATURE

Latin symbols		Unit
D_B	Brownian diffusion coefficient	m^2/sec
D_T	Thermophoretic diffusion coefficient	m^2/sec
$\vec{q}$	Fluid velocity	m/sec

Latin symbols		Unit
d	Dimensional layer depth	m
$\vec{g}$	Acceleration due to gravity	
R_m	Basic density Rayleigh number, $Rm = \frac{[\rho_p\phi_0+\rho(1-\phi_0)]gd^3}{\mu\kappa_T}$	
R_n	Concentration Rayleigh number, $R_n = \frac{(\rho_p-\rho)(\phi_1-\phi_0)gd^3}{\mu\kappa_T}$	
Le	Lewis number, $Le = \frac{\kappa_T}{D_B}$	
Ta	Taylor number, $Ta = \left(\frac{2K\Omega K}{\nu\varepsilon}\right)^2$	
Pr	Prandtl number, $Pr = \frac{\mu}{\rho\kappa_T}$	
(x^*, y^*, z^*)	Cartesian coordinates	m
N_A	Modified diffusivity ratio, $N_A = \frac{D_T(T_h-T_c)}{D_BT_c(\phi_1-\phi_0)}$	
N_B	Modified particle-density increment, $N_B = \frac{(\rho c)_p}{(\rho c)_f}(\phi_1-\phi_0)$	
$\vec{g}$	Modulated gravity field	m/sec^2
$\vec{q}$	Nanofluid velocity	m/sec
p	Pressure	kg/(m sec^2)
T	Temperature	K
T_h	Temperature at the lower wall	K
T_c	Temperature at the upper wall	K
k_m	Thermal conductivity	W/mK
Ra	Thermal Rayleigh number, $Ra = \frac{\rho g\beta d^3(T_h-T_c)}{\mu\kappa_T}$	
τ	Time	sec
Greek symbols		
$(\rho c)_p$	Effective heat capacity of the nanoparticle material	J/kgK
ρ_f	Fluid density	kg/m^3
ω	Frequency of modulation	sec^{-1}
$(\rho c)_f$	Heat capacity of the fluid	J/kgK
α	Horizontal wave number	
ρ_p	Nanoparticle mass density	kg/m^3
$\vec{\Omega}$	Rotational speed vector	
β	Proportionality factor	K^{-1}
μ	Fluid Viscosity	mPa sec
κ_T	Effective thermal diffusivity of the fluid	m^2/sec

Latin symbols		Unit
ν	Kinematic viscosity μ/ρ_f	m^{4}Pa sec/kg
ϕ	Nanoparticle volume fraction	
ψ	Stream function	m^2/sec
δ	Amplitude of modulation	
Subscripts		
b	Basic	
Superscripts		
$*$	Dimensional variable	
$'$	Perturbation variable	
Operators		
∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$	

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