



Critical Speed Analysis of Rotor Shafts Using Campbell Diagrams

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Abstract

The main aim of this paper is to avoid the critical speed at low rotational velocities for three different cases, i.e. shaft without rotor, single rotor system and two rotor system. The critical speeds of these rotor systems are analyzed with two boundary conditions, viz. one end supported, both ends supported. Moreover, the rotors are mounted at two different positions: single rotor is placed at middle of the shaft and the same rotor is split into two halves and kept at equal distance from the either end of shaft. This critical speed analysis is carried out on both solid and hollow shafts. The range of rotational speed for the analyses considered is between 0 to 5000 rpm. The critical speeds of various rotor systems are studied using Campbell diagram and it is observed that, the critical speeds are altered by changing the boundary conditions and replacing the solid shaft with hollow shaft of same torsional stiffness as well.

Keywords : Campbell diagram, Natural frequency, Critical speed, Modes, Torsional stiffness.

I. Introduction

The major problem for occurrence of resonance in rotating shafts is matching of rotational frequency to the natural frequency of the system. When the external frequencies come close to the systems frequency, high amplitudes are generated at low speeds. These speeds are known as critical speed, which can be defined as the theoretical angular velocity that excites the natural frequency of the rotating shafts. The shaft diameter and length are kept constant. The rotor diameter and lengths are changed. These are changed in a way that the center of gravity is not disturbed. Modelling is done in NX11 software and the critical speed analysis is carried out in ANSYS Workbench software.

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II. Purpose & Scope

The purpose of the project is to alter the critical speed of rotating members. The critical speed is altered based on the rating of the motor. This is to avoid the initial critical speed which occurs at low speeds. Hence the resonance condition can be avoided for the entire system. Two boundary conditions are taken for all shafts, i.e. the shafts are one end supported and both ends supported. The shaft is modelled in Unigraphics NX11 and the volume, mass and weight are obtained, then imported to Ansys Workbench and performed vibration analysis. The obtained output i.e. the natural frequency is compared with the analytical results. The rotor is attached at the centre of the shaft, the same rotor is split into two halves and attached at equal distance from the ends of the shaft. The solid shaft is replaced with the hollow shaft of same torsional stiffness and the critical speed analysis is done and compared all the results.

III. Literature

Based on the analysis of a Jeffcott rotor-bearing model, it is found that the condition for the response to occur is strongly dependent on stiffness of the supports, Lyn M. Greenhill et al.[VI] The first resonant speed will be higher than the rated rotational speed theoretically, and empirically needs more than 20 percent margin i.e. 120 percent rule for nonlinear spring mass model, Shuji Tanaka et al.[XI] Evaluation of all stiffness and damping coefficients of wear rings, mechanical seals, throttle seals can help to avoid resonance in between 25 to 125 percent of operating speed for a multistage centrifugal pump Harisha S. et al.[III] An open crack on the shaft in the rotor system changes its stiffness. The effect of which is identified on natural frequency of the system. The natural frequency of the cracked rotor increases in comparison to uncracked rotor, R. Tamrakar et al.[X]

IV. Problem Formulation

All rotating shafts even in the absence of external loading will deflect due to rotation. The range of deflection depends upon the boundary condition, stiffness of the shaft, mass and loading if any. When the external frequency matches or comes close to the natural frequency of the system resonance occurs. The speed at that frequency is known as critical speed. Critical speed can be defined as the theoretical angular velocity which excites the natural frequency of the system. The unbalance mass causes deflection that creates resonance. The natural frequency of the system is calculated first and the critical speeds are analyzed through Campbell diagram, which shows the interference between frequency in hertz to rotational velocity in rad/sec. Fundamental frequencies of shafts calculated for different boundary conditions.

Shaft with One End Supported

For the uniform diameter shaft supported at one end, the fundamental natural frequencies at various modes can be calculated as follows; $f_n = \frac{\pi}{2} \left(n - \frac{1}{2} \right) 2\sqrt{gEI/wL^4}$
Where n is the mode and must be an integer 1, 2, 3.....
 g is acceleration due to gravity in m/s^2

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E is Young's modulus of structural steel (7800 Kg/m³)

I is moment of inertia in m⁴,

w is load in Newtons and

L is length of the shaft in meters.

Note: Mode is the fundamental natural frequency of the system or shaft at any instant.

Shaft with Both Ends Supported

For the uniform diameter shaft supported at both ends, the fundamental natural frequencies at various modes can be calculated as follows; $f_n = \frac{\pi}{2} \left(n + \frac{1}{2} \right) 2\sqrt{gEI/wL^4}$

Hollow Shaft of Same Torsional Stiffness as Solid Shaft

The diameter of the solid shaft (d) taken as 100 mm. In case of hollow shaft it is assumed $d_o = 2*d_i$, where d_o and d_i are the outer and inner diameter respectively. For same material, mass and torsional strength, outer and inner diameters of the hollow shaft are calculated.

$$J = \frac{\pi}{32} (d^4) = \frac{\pi}{32} (d_o^4 - d_i^4), \quad \text{Results} \quad d_o = 101.626 \text{ mm} \quad \& \quad d_i = 50.813 \text{ mm}$$

Materials

The material chosen for the models is Structured Steel as its density and percentage of carbon is high. Due to high loading, we may get a greater number of critical speeds which can be predicted. The material is selected just to show how the critical speed is varying for different boundary conditions and loading and by what percentage the range of critical speed can be altered.

V. Results and Discussion

Case I : Uniform Solid Shaft (Analytical vs Simulation)

A model of uniform shaft is created in Ansys workbench with the structured steel. The length of the shaft in all cases is taken as 1000 mm. The diameter of the solid shaft is taken as 100 mm. Modal analysis is done for the shaft with different boundary conditions. Rotational speed range is taken from 0 to 5000 rpm for all models and all boundary conditions.

The Campbell diagram plotted after the simulation for the solid shaft with one end supported. Listed all modal frequencies in the given range of rotational speed. Compared the analytical and simulation modal frequencies.

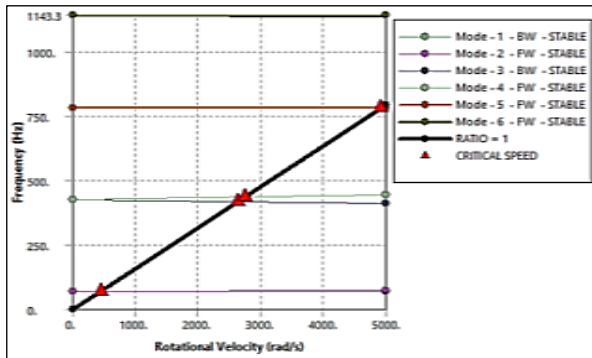


Figure 1: Campbell Diagram for Shaft with One End Supported

Table 1: Modal frequencies (Hz) for solid shafts

Mode	Analytical	Simulation
M1	71.3989	70.508
M2	446.6277	427.77
M3	789.8115	782.59

For the solid shaft with both ends supported variation of the critical speed with respect to frequency and rotational velocity are given by Campbell diagram as shown below. Compared the analytical and simulation modal frequencies.

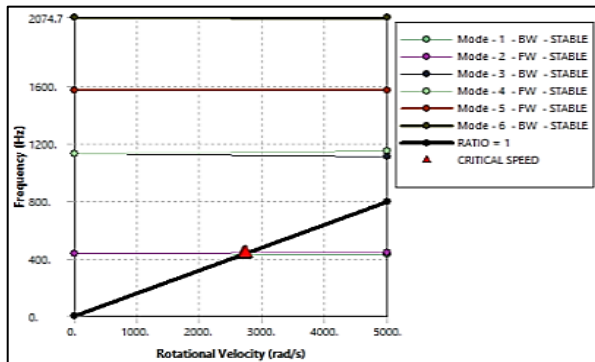


Figure 2: Campbell Diagram of Shaft with Both Ends Supported

Table 2: Modal frequencies (Hz) for solid shafts

Mode	Analytical	Simulation
M1	446.6277	432.28
M2	1240.632	1127.9
M3	2431.63	2074.7

Case II : Uniform Solid Shaft with Single Rotor

A model of uniform shaft with single rotor is created in Ansys workbench with the structured steel. The length and diameter of the shaft are 1000 mm and 100 mm respectively. Modal analysis is done for the shaft with different boundary conditions. The rotor of length 100 mm and diameter 200 mm is placed exactly at the middle of the shaft.

The Campbell diagram in Figure 3 shows the frequencies and corresponding critical speeds are recorded for the solid shaft with centre rotor and one end supported and compared with similar cases.

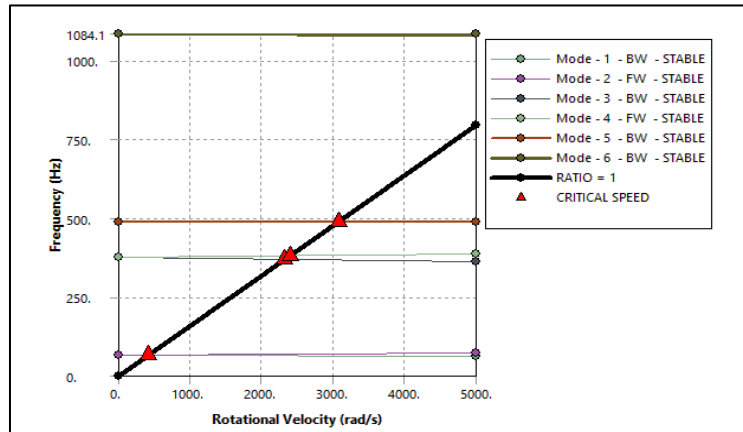


Figure 3: Campbell Diagram for Solid Shaft with Centre Rotor and One End Supported

The critical speed analysis is done for the solid shaft with centre rotor and both ends supported. Within the range of speed all the available critical speeds are recorded with respect to frequency variation.

Case III : Uniform Solid Shaft with Two Rotors

Modal analysis is done for the shaft of length 1000 mm and diameter 100 with two rotors and different boundary conditions. Two identical rotors of length 50 mm and diameter 200 mm are placed at a distance of 225 mm from both the ends.

For the solid shaft with two rotors and one end supported, the Campbell diagram gives the frequencies and corresponding critical speeds. Listed all the available critical speeds within the range.

The critical speed analysis is done for the shaft with two rotors and both ends supported and all the critical speeds are recorded with respect to frequency variation.

Case IV : Hollow Shaft of Same Torsional Stiffness as Solid Shaft

The material is removed from the solid shaft (case I) using an axial hole. The assumption for hollow shaft is, the inner diameter is half of outer diameter. Torsional stiffness is maintained same for both cases. The internal diameter (d_i) = 50.813 mm and external diameter (d_o) = 101.626 mm are taken as per the calculation shown in section IV.

The modal frequencies and corresponding whirling speeds found from simulation (Campbell diagram) are recorded for uniform hollow shaft with one end supported and compared with all similar cases in Observations Section.

After the simulation plotted the Campbell diagram for hollow shaft with both ends supported as shown in Figure 4. Listed all critical speeds and modal frequencies in the given range of rotational speed.

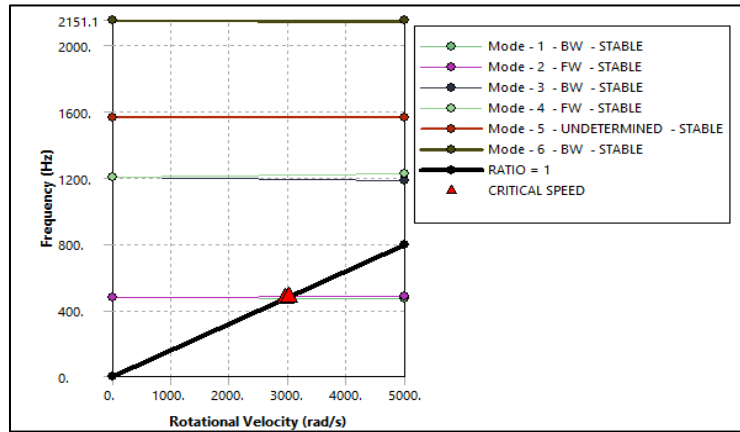


Figure 4: Campbell Diagram for Hollow Shaft with Both Ends Supported

Case V : Uniform Hollow Shaft with Single Rotor

A hollow shaft model is created with outer diameter (d_o) = 101.626 mm and inner diameter (d_i) = 50.813 mm in Ansys workbench with the structured steel. A rotor is of length 100 mm and diameter 200 mm and made of same material is attached at the middle of the shaft. Modal analysis is done for the shaft with different boundary conditions.

The modal frequencies and corresponding critical speeds are recorded from the Campbell diagram for hollow shaft with centre rotor and one end supported and compared all critical speeds with other similar cases.

Listed the variation of critical speeds with respect to frequency and rotational velocity from Campbell diagram as shown in Figure 5 for hollow shaft with one rotor and both ends supported.

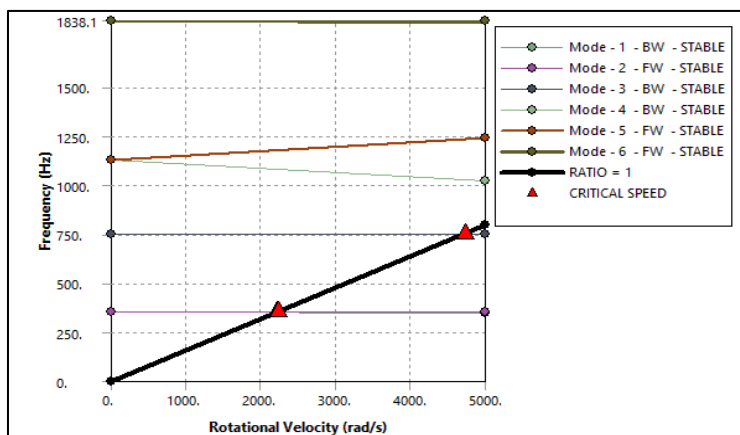


Figure 5: Campbell Diagram for Hollow Shaft with One Rotor and Both Ends Supported

Case VI : Uniform Hollow Shaft with Two Rotors

A model of hollow shaft with two rotors is created in Ansys workbench with the structured steel. Modal analysis is done for the hollow shaft with different boundary conditions. Two identical rotors are placed as per case III.

The Campbell diagram plotted after modal analysis and the frequency and critical speed are recorded for hollow shaft with two rotors and one end supported. Compared the results with similar cases.

The variation of the critical speed with respect to frequency and rotational velocity are listed from Campbell diagram for hollow shaft with two rotors and both ends supported.

Observations

Plotted the critical speeds and corresponding modes of vibrations for solid shafts and hollow shafts with end supported conditions. Within the rotational speed limit, it is found 5 critical speeds for all six cases with one end support and found 2 to 3 critical speeds for solid shafts and hollow shafts with both ends support. It is observed from Figure 6 and Figure 7, the critical speed value is high corresponding to higher modes for both the solid and hollow shafts without rotor when compared to rotor-shafts with one end support. Whereas, such kind of phenomenon is not observed for the shafts with both ends supported.

For both ends support cases, within the speed range of 0 to 5000 rpm, shafts with single rotor have 3 critical speeds, whereas shafts without rotor and shafts with two rotors have only 2 critical speeds as shown in Figure 8 and Figure 9.

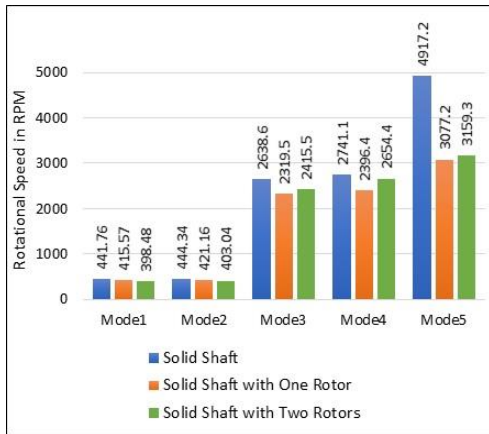


Figure 6: Critical Speeds of Solid Shafts with One End Supported

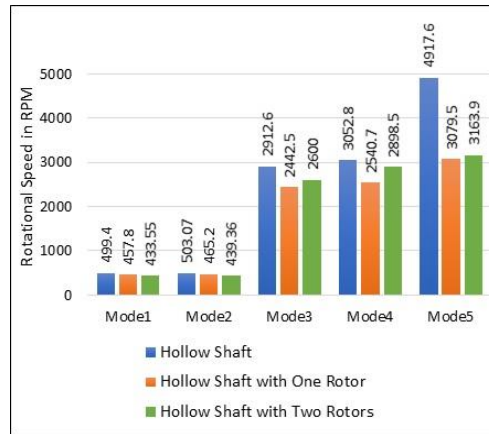


Figure 7: Critical Speeds of Hollow Shafts with Both Ends Supported

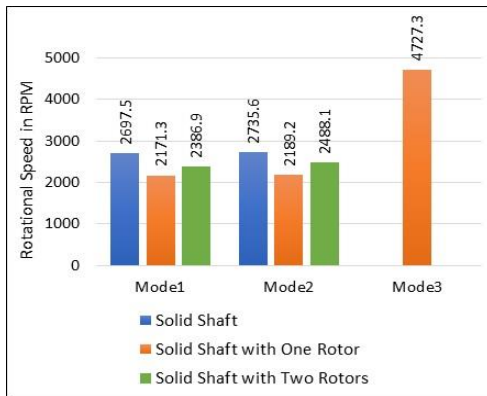


Figure 8: Critical Speeds of Solid Shafts with Both Ends Supported

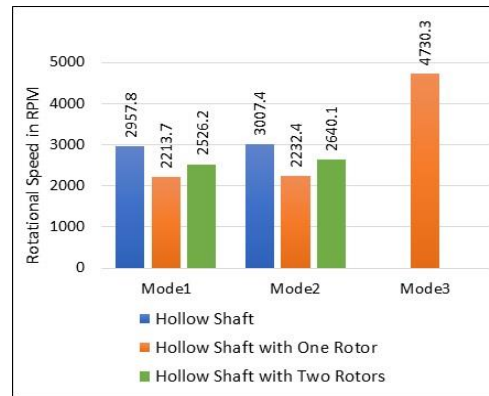


Figure 9: Critical Speeds of Hollow Shafts with Both Ends Supported

VI. Conclusion

The dynamics behavior of various rotor systems is studied using Campbell diagram. The critical speeds of these systems are simulated and from the results the following conclusions are made:

- Critical speeds can be altered by changing the boundary conditions, replacing the solid shaft with hollow shaft of same torsional stiffness or by distributing the same mass into two equal halves, placed at equal distance from the ends of the shaft.
- The critical speed can be shifted to some higher value by replacing the solid shaft with the hollow shaft which possess same torsional stiffness, e.g. for single rotor shaft with both ends supported case the resonance speed is found to be increased from 2171.3 rad/s to 2213.7 rad/s.
- For the shaft with both ends supported by redistributing the mass, the range of critical speed is increased from 2171.3 rad/s to 2386.9 rad/s.

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